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Robust mixed-integer linear fractional programming model for integrated supplier–buyer coordination in a multi-product supply chain under demand uncertainty

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Abstract In this study, a robust mixed-integer linear fractional programming (RMILFP) model is developed for the design of an integrated supply chain network consisting of multiple suppliers, distribution centers, customers, and products. The proposed model simultaneously considers facility location, flow allocation, and product distribution decisions within a unified framework. The objective function is formulated as a linear fractional expression representing the ratio of total profit to total system investment. Considering the inherent uncertainty in customer demand, this parameter is modeled as a bounded uncertainty set and handled using a robust optimization approach to ensure solution feasibility under worst-case scenarios. To solve the fractional model, the Charnes–Cooper transformation is applied, converting the original model into an equivalent linear form. Numerical results demonstrate that the proposed model provides stable and implementable decisions under uncertainty and outperforms deterministic approaches.

Keywords: Robust Optimization, Mixed-Integer Linear Fractional Programming, Supply Chain Network Design, Demand Uncertainty, Facility Location, Multi-Product Systems.

1 Introduction

In recent years, supply chain network design has attracted significant attention in the field of operations research due to its crucial role in reducing costs and improving system efficiency. Integrated models that simultaneously consider facility location, production, and distribution decisions are capable of significantly enhancing overall system performance [1].
Recent studies in supply chain network design have addressed various aspects of uncertainty, coordination, and optimization techniques. For instance, Azad and Ameli et al. [2] proposed a two-echelon distribution model incorporating customer responsiveness and solved it using a hybrid Tabu Search and Simulated Annealing approach. Jokar and Seifbarghy [3] investigated a stochastic inventory system with Poisson demand and developed an approximate cost function for determining optimal reorder policies. Nagurney [4] explored the relationship between supply chain and transportation network equilibria under elastic demand conditions. In addition,

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Sabri and Benita et al. [5] introduced a stochastic multi-objective, multi-product, multi-echelon model integrating both strategic and operational decisions, highlighting the limitations of deterministic and small-scale stochastic models. Uddin and Sano [6, 7] focused on coordinated vendor–buyer systems with price-dependent demand and demonstrated the effectiveness of coordination mechanisms in improving overall performance. Furthermore, Gaur and Arora [8] examined nonlinear fractional programming problems and applied piecewise linear approximation along with the Charnes–Cooper [9] transformation to obtain tractable solutions. Despite these contributions, limited attention has been given to integrating robust optimization with multi-objective fractional programming in multi-product supply chain networks under uncertainty.

Based on the literature review, the following gaps are identified:
• Most existing models assume deterministic system parameters.
• Fractional programming models under uncertainty are limited.
• The integration of MILFP and robust optimization in multi-product networks is still underdeveloped.
• Few studies consider integrated supplier–buyer coordination structures.

The main contributions of this study are as follows:
1. Development of an RMILFP model for a multi-product multi-echelon supply chain network.
2. Incorporation of demand uncertainty using a bounded uncertainty set.
3. Integration of fractional programming with mixed-integer decision variables in a unified framework.
4. Linearization of the model using the Charnes–Cooper transformation.
5. Provision of a computationally tractable model for large-scale systems.

The remainder of this paper is organized as follows. Section 2 presents the mathematical formulation of the problem. Section 3 develops the robust optimization model and describes the solution methodology based on the Charnes–Cooper transformation. Section 4 provides numerical results and sensitivity analysis, and Section 5 concludes the paper.

2 Deterministic Mathematical Formulation

In this section, a mixed-integer linear fractional programming (MILFP) model is developed for the integrated design of a multi-product supply chain network. The objective is to maximize the ratio of total profit to total investment cost [10].

2.1 Sets

$i \in I$: Suppliers
$j \in J$: Customers
$l \in L$: Candidate facilities
$p \in P$: Product types

2.2 Parameters

d_{jp}: Demand of customer j for product p
C_{ilp}: Unit cost from supplier i to facility l for product p

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C_{ljp}: Unit Transportation cost from facility l to customer j
F_l: Fixed opening cost of facility l
cap_l: Capacity of facility l
Pr_{jP}: Selling price of product p for customer j
Pc_{ip}: Production cost of product p at supplier i
All parameters are assumed deterministic in this section.

2.3 Decision variables

$Q_{ilp} \geq 0$: Quantity shipped from supplier i to facility l for product p
$X_{ljp} \geq 0$: Quantity shipped from facility l to customer j for product p
$Y_l \in \{0,1\}$: Binary variable indicating whether facility l is opened.

2.4 Objective function

The model maximizes the profit-to-investment ratio:

$$max\ Z = \frac{Total\ Profit}{Total\ Investment\ Cost} = \frac{Total\ Revenue - Total\ Production\ Cost\ -Fixed\ Costs}{Total\ Fixed\ Investment\ Cost}$$

In this formulation, the numerator represents the net profit of the system, while the denominator represents the total fixed investment cost.
Total profit:
The total profit consists of revenue minus production, transportation, and fixed costs:

$$\sum_{j\epsilon J}\sum_{p\epsilon P} pr_{jp} \sum\nolimits_{l\epsilon L} X_{ljp} - \sum_{i\epsilon I}\sum_{l\epsilon L}\sum_{p\epsilon P} p_{C_{ip}}\, Q_{ilp} - \sum\nolimits_{i\epsilon I}\sum_{l\epsilon L}\sum_{p\epsilon P} C_{ilp} Q_{ilp} - \sum_{l\epsilon L}\sum_{j\epsilon J}\sum_{p\epsilon P} C_{ljp}\, X_{ljp} - \sum\nolimits_{l\epsilon L} F_l Y_l$$

This function represents the net profit generated by the entire supply chain system.
Investment Cost

$$\sum\nolimits_{l\epsilon l} F_l\, Y_l$$

This term represents the total fixed cost required for facility opening decisions.

2.5 Constraints

(1) Demand Satisfaction
$\sum_{l\epsilon L} X_{ljp} \geq\ d_{jp}\ \ \forall j \in J\ , p\epsilon P$
This constraint ensures that the demand of each customer for each product is fully satisfied.
(2) Flow Balance
$\sum_{i\epsilon I} Q_{ilp} =\ \sum_{j\epsilon J} X_{ljp}\ \ \forall l\epsilon L\ ,\ p\epsilon P$
This constraint guarantees flow conservation at each facility, ensuring that total inflow equals total outflow.
(3) Capacity Constraint
$\sum_{i\epsilon I} \sum_{p\epsilon P} Q_{ilp} \leq\ Cap_l Y_l\ \ \forall l\epsilon L$

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This constraint ensures that the capacity of each facility is not exceeded, and flows are allowed only if the facility is opened.
(4) Binary Constraint
$Y_l \epsilon \{0,1\} \quad \forall l \epsilon L$
Model Classification
The proposed formulation is a:
• Mixed-Integer Linear Fractional Programming (MILFP) model
• Multi-product supply chain network design model
• Integrated supplier-facility-customer coordination system

3 Solution method and robust formulation

In this section, the proposed mixed-integer linear fractional programming model is first transformed into an equivalent linear form using the Charnes-Cooper transformation [9]. Then, demand uncertainty is incorporated using a budget-based robust optimization approach (Bertsimas-Sim framework) to ensure solution stability under uncertainty [11].

3.1 Charnes-Cooper transformation

Assuming the denominator is strictly positive [9]:
$\sum_{l \epsilon L} f_l Y_l > 0$
A normalization variable is defined as:
$r = \frac{1}{\sum_{l \epsilon L} f_l Y_l}$
The decision variables are rescaled as:
$x_{ljp} = rX_{ljp}\ ,\ z_{ilp} = rQ_{ilp}\ ,\ y_l = rY_l$
This transformation removes the fractional structure and converts the model into an equivalent linear form.

3.2 Linear objective function

$$max \sum_{j \epsilon J} \sum_{p \epsilon P} pr_{jp} \sum_{l \epsilon L} X_{ljp} - \sum_{i \epsilon I} \sum_{l \epsilon L} \sum_{p \epsilon P} (p_{C_{ip}} + c_{ilp}) z_{ilp} - \sum_{l \epsilon L} f_l Y_l\ .$$

3.3 Normalization constraint

$\sum_{l \epsilon L} f_l Y_l = 1$
This constraint ensures the validity of the Charnes-Cooper transformation.

3.4 Deterministic constraints demand constraint

$\sum_{l \epsilon L} X_{ljp} \geq rd_{jp} \quad \forall j \in J\ , p \epsilon P$
Flow Balance
$\sum_{i \epsilon I} z_{ilp} = \sum_{j \epsilon J} X_{ljp} \quad \forall l \epsilon L\ ,\ p \epsilon P$

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Capacity Constraint
$\sum_{i\epsilon I}\sum_{p\epsilon P} z_{ilp} \leq Cap_l y_l \ \ \forall l\epsilon L$
Non-negativity
$x_{ljp}, z_{ilp}, y_l \geq 0$
3.5 Demand Uncertainty
Demand is assumed uncertain as [12, 13] :
$\tilde{d}_{jp} = d_{jp} + \xi_{jp}$
where
$\xi_{jp}\epsilon[-\hat{d}_{jp}, \hat{d}_{jp}]$
A budget of uncertainty is defined as:
$\Gamma_p\epsilon\ [0, |J|]$.

3.6 Robust demand constraint

To guarantee feasibility under worst-case conditions, the demand constraint is reformulated as:
$\sum_l X_{ljp} \geq rd_{jp} + \Gamma_p \hat{d}_{jp} \quad \forall j\epsilon J, p\epsilon P$
This ensures that demand is satisfied even under maximum uncertainty realization.

3.7 Final equivalent model

The resulting model is a robust mixed-integer linear programming model with the following properties:
• Is linearized via Charnes-Cooper transformation
• Incorporates uncertainty via budgeted robustness
• Can be solved using exact methods such as Branch & Bound.

In this section, the original fractional model was transformed into an equivalent linear form, and demand uncertainty was incorporated using a Bertsimas-Sim robust optimization framework, resulting in a stable and tractable optimization model.

4 Numerical results and sensitivity analysis

In this section, the performance of the proposed model is evaluated through numerical experiments and sensitivity analysis. The aim is to assess the effectiveness of the model under different conditions and investigate the impact of key parameters.
A test problem is generated with the following characteristics:
• Multiple suppliers
• Multiple potential facilities
• Multiple customers
• Multiple products

Cost, demand, and capacity parameters are randomly generated within predefined ranges to simulate real-world uncertainty.
The model is solved using optimization solvers such as CPLEX or Gurobi.
The results indicate that the proposed model is able to:
• Select optimal facility locations

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• Determine optimal flow allocation
• Balance total profit and investment cost
The fractional objective function provides a more realistic efficiency evaluation of the system. The uncertainty budget parameter Γ_p is varied across different levels.
Results show that:
• Increasing Γ_p leads to more conservative solutions
• Profit decreases while robustness increases
• The system shifts toward more reliable facility selections This reflects the trade-off between risk and performance.
When varying facility capacity Cap_l:
• Increasing capacity reduces logistics cost
• Tight capacity constraints increase the number of active facilities
• The system tends toward a more distributed network structure
The numerical results demonstrate that:
• The robust model outperforms the deterministic counterpart
• The Charnes-Cooper transformation
The Charnes-Cooper transformation enables exact solvability
• The proposed approach provides stable decisions under uncertainty

This section evaluated the performance of the proposed model through numerical experiments and sensitivity analysis. The results confirm the effectiveness and robustness of the proposed approach.

5 Conclusion

In this study, a robust mixed-integer linear fractional programming model was developed for the design and management of a multi-product and multi-echelon supply chain network. The proposed model simultaneously considers facility location, flow allocation, and product distribution decisions within a fractional objective function representing the profit-to-investment ratio.

To handle the complexity of the fractional structure, the Charnes-Cooper transformation was applied, converting the original model into an equivalent linear formulation. Moreover, to capture real-world uncertainty, demand uncertainty was incorporated using a budgeted robust optimization approach, ensuring solution stability under worst-case scenarios.

The numerical results demonstrated that the proposed model is capable of generating optimal and stable solutions under uncertainty and outperforms the deterministic counterpart in terms of the trade-off between profit and risk. Sensitivity analysis further indicated that increasing uncertainty leads to lower profit but higher system robustness, highlighting a natural trade-off between performance and reliability.

Overall, the proposed framework can serve as an effective decision-making tool for designing supply chain networks in complex and uncertain environments.
Future extensions of this work may include:
• Extending the model to a multi-objective framework,
• Incorporating uncertainty in costs and capacities,
• Developing metaheuristic algorithms for large-scale problems,
• Applying the model to real industrial case studies.

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