International Journal of Applied Operational Research
Vol. 14, No. 4, pp. 21-38, Autumn 2026

Journal homepage: ijorlu.liau.ac.ir

Efficient battery siting in distribution networks using SBM-DEA

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Received: 22 July 2026 ; **Accepted:** 10 September 2026

Abstract Battery energy storage planning in distribution networks is inherently multi-criteria. The alternative that produces the largest peak reduction may be excessively costly; the minimum-loss alternative may deliver insufficient peak support; and a charging schedule that is beneficial at one bus may aggravate local voltage drop at another. This paper develops an integrated framework for evaluating 63 battery siting and sizing alternatives on the IEEE 33-bus distribution system. In the first stage, hourly AC load flow is performed for every alternative to calculate investment cost, peak reduction, discharged energy, daily energy losses, and voltage deviation. In the second stage, the alternatives are treated as homogeneous decision-making units and evaluated using a non-radial undesirable-output slacks-based measure data envelopment analysis model. Investment cost is the input; peak reduction and discharged energy are desirable outputs; and energy losses and voltage deviation are undesirable outputs. SBM is selected for three substantive reasons: technical and economic criteria contain independent slacks, criterion changes are non-proportional, and undesirable outputs must be modeled directly without reciprocal or translation transformations. Super-SBM is then used to discriminate among efficient alternatives. Under the defined 63-alternative set and the stated deterministic assumptions, a 750-kW, four-hour battery at bus 14 obtains the highest super-efficiency score; this result identifies the most efficient observed alternative within the study design and should not be interpreted as a universally optimal network-planning solution. Nevertheless, the minimum-loss alternative ranks only 35th, demonstrating that superior single-criterion performance is not equivalent to efficient capital use. Sensitivity analyses on battery cost structure, PV penetration, loading level, and returns to scale show that the ranking pattern is broadly stable, although the best location changes with operating conditions. The framework is therefore suitable for technical-economic screening before detailed stochastic or robust investment optimization.

Keywords: Data Envelopment Analysis, SBM, Undesirable Outputs, Battery Energy Storage, Distribution Network, IEEE 33-Bus, Load Flow.

[DOI: 10.71885/ijorlu-2026-4-743]

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1 Introduction

This section formulates the planning problem from the distribution-system operator perspective, identifies the limitations of conventional assessment methods, and states the research gap and contributions. The methodological choice is therefore derived from the decision problem rather than from a superficial combination of power-system analysis and DEA.

Battery planning involves three connected decision layers: device ratings, electrical location, and operating schedule. Treating these layers as fully independent can select an alternative that appears economical but creates unacceptable voltage or loss conditions during particular hours. The assessment framework must therefore retain conflicting economic and technical information rather than compressing it prematurely into a single weighted objective.

The expansion of distributed generation, rapid photovoltaic deployment, flexible demand, and active distribution-network operation has transformed battery planning from a simple equipment-selection problem into a multilayer planning decision. A battery can shift energy from low-load or high-renewable periods to evening peaks, reduce upstream demand, defer network reinforcement, and improve losses and voltage when properly dispatched. The same device can, however, increase branch current and aggravate local voltage drop when it is located or charged inappropriately. Consequently, the statement that battery installation improves network performance is scientifically incomplete. The relevant question is which combination of location, power rating, and energy duration produces the most balanced portfolio of benefits for the capital committed.

Most battery energy storage system (BESS) planning studies optimize an economic, technical, or weighted composite objective. A single objective suppresses relevant trade-offs, whereas weighted multi-objective formulations require externally specified preferences. Costs, kilowatts, kilowatt-hours, and per-unit-hours cannot be aggregated without value judgments. DEA offers a useful complementary perspective because it compares homogeneous alternatives with multiple inputs and outputs without imposing a parametric production function.

DEA has been extensively applied to electricity utilities, generation plants, regions, and time periods, but considerably less attention has been given to engineering design alternatives. Here, a DMU is not an organization; it is a battery decision defined by location, power, and duration. Homogeneity is maintained because every alternative is evaluated on the same feeder, horizon, load and PV profiles, and dispatch logic.

Losses and voltage deviation require particular care. They are not resources consumed by the alternative and should not be inserted as inputs merely to preserve a conventional DEA structure. Reciprocal or translation transformations are also undesirable because results may depend on arbitrary constants. They are therefore represented directly as undesirable outputs in an SBM model.

The paper contributes an auditable load-flow-to-DEA chain, 63 homogeneous siting-sizing-duration alternatives, component-based battery cost, direct treatment of two technical bad outputs, an explicit methodological justification for SBM, Super-SBM discrimination, physical interpretation of location effects, and sensitivity analysis on cost, PV, load, and returns to scale. Each numerical result is interpreted through current paths, charging and discharging timing, power-to-energy ratio, and capital opportunity cost rather than being reported as an isolated efficiency score.

[DOI: 10.71885/ijorlu-2026-4-743]

2 DEA in power systems and energy-storage planning: literature review

This review does more than catalogue previous studies. It traces the methodological development of DEA, examines how efficiency analysis has been used in electricity-sector regulation and planning, reviews the technical and economic criteria adopted in battery siting and sizing, and identifies the unresolved link between hourly network simulation and relative-efficiency assessment. This structure is necessary because the present problem lies at the intersection of operations research, distribution-system engineering, and investment appraisal.

Optimization and DEA answer different decision questions. Optimization searches for a feasible solution that minimizes or maximizes a predefined objective, while DEA evaluates how efficiently each observed alternative converts resources into desirable services while limiting adverse consequences. In battery planning, an optimization model can generate candidate locations, sizes, and schedules, but its result depends on the selected objective coefficients and economic weights. DEA is therefore used here as a post-simulation benchmarking layer, not as a substitute for AC load flow, operational scheduling, or feasibility analysis. The distinction prevents the efficiency score from being interpreted as a physical operating optimum.

The methodological stream begins with the CCR model, which measures overall technical efficiency under constant returns to scale, and the BCC model, which separates pure technical efficiency from scale effects [1,2]. Both models are radial: they seek proportional contraction of inputs or proportional expansion of outputs. Such proportionality is restrictive for battery alternatives because investment cost, peak reduction, discharged energy, loss, and voltage deviation do not improve at a common rate. Tone's SBM measures inefficiency through the average normalized slacks of individual variables and therefore identifies the specific sources of inefficiency that remain hidden in radial scores [3]. Super-SBM subsequently permits discrimination among efficient units [4]. The undesirable-output literature further shows that emissions, losses, service interruptions, and quality deterioration should be modeled as adverse products of the production process rather than being relabeled as conventional inputs [5-9]. This methodological development directly supports the formulation adopted in the present paper.

The broader DEA literature confirms that model choice should follow the structure of production technology rather than convenience. Standard DEA texts and retrospective surveys document the progression from radial efficiency measurement to slack-sensitive, environmental, stochastic, network, and ranking extensions [6-12]. In particular, the treatment of undesirable outputs has been formalized through alternative disposability assumptions and non-radial formulations [5-9], while stochastic super-efficiency has been applied to electricity-distribution companies when observed data are uncertain [13]. These developments support the present use of a non-radial undesirable-output specification, while also clarifying that stochastic DEA is a distinct extension and is not claimed in the deterministic base model used here.

Related methodological contributions by the authors and other DEA researchers further illustrate how ranking and benchmarking can be adapted when standard DEA assumptions are insufficient. Zanboori et al. developed a feasible SBM-based super-efficiency ranking model [14], extended ranking to interval data [15], and contributed inverse-DEA formulations for restructuring, negative data, and time-dependent assessment windows [16-18]. These studies are not battery-planning models; their relevance here is methodological, particularly for discrimination among efficient units and for interpreting DEA as a relative benchmarking technology. In applied performance assessment, Karimi and co-authors used DEA for

[DOI: 10.71885/ijorlu-2026-4-743]

organizational-agility evaluation and for financial-performance evaluation in the presence of negative data [19, 20], further illustrating the importance of matching DEA formulation to the sign, meaning, and production role of the variables.

A second stream applies DEA to electricity generation, transmission, and distribution. Early regulatory studies used DEA to benchmark utilities with inputs such as labor, operating expenditure, network length, transformer capacity, and capital, and outputs such as delivered energy, customer numbers, and service coverage [21-23]. The main economic purpose was to estimate relative cost efficiency, identify attainable operating targets, and support incentive regulation. Energy and environmental studies later extended the framework by including fuel use, emissions, outage measures, and quality indicators, thereby recognizing that an electricity system may produce valuable energy and undesirable technical or environmental effects simultaneously [24-27]. These studies establish the relevance of DEA for heterogeneous multi-input-multi-output systems, but their DMUs are usually firms, plants, regions, or time periods. They rarely treat a physical network-design alternative as the decision-making unit.

The electricity-sector DEA literature also clarifies two modeling issues that are essential in this study. First, variable selection must follow the production logic of the system rather than the mere availability of data [28]. Adding several correlated indicators may create artificial efficiency because each additional variable gives DMUs another route to the frontier. Second, returns to scale must reflect the decision context. Under CRS, an oversized battery can be penalized for scale inefficiency because capacity itself is a planning choice; under VRS, the model asks whether an alternative is well configured relative to other alternatives of a similar effective scale. For this reason, CRS is used for the main ranking and VRS is retained as a robustness test rather than as an interchangeable specification.

Variable-selection and model-structure studies provide an additional caution for the present application. DEA discrimination can deteriorate when too many weakly informative indicators are introduced; dedicated variable-selection procedures have therefore been proposed to improve discriminatory power [28]. Network DEA, including the network SBM formulation, is useful when internal subprocesses and intermediate products are explicit decision objects [29,30]. The present study intentionally uses a black-box alternative-level DEA because hourly charging, discharging, and power-flow calculations are resolved before benchmarking; a future network-DEA extension could instead expose these internal stages as linked subprocesses.

The third-stream concerns BESS planning in distribution networks. Major reviews identify lifecycle cost, capital expenditure, energy arbitrage, peak shaving, loss reduction, voltage regulation, renewable-energy accommodation, reliability, and upgrade deferral as the most common planning criteria [31-35]. The technical literature uses AC or linearized power flow together with mixed-integer programming, convex optimization, dynamic programming, genetic algorithms, particle swarm optimization, and Pareto-based methods [36-42]. These studies demonstrate that location and size cannot be evaluated from nameplate ratings alone: a unit installed farther from the substation may reduce current over more upstream branches during discharge, yet it may also intensify voltage drop during charging. Similarly, increasing duration may improve energy shifting but can produce diminishing marginal technical value once the evening peak is already covered.

Recent work also demonstrates that DEA can be used to compare energy-storage technologies across multiple sustainability dimensions [43], whereas distribution-network planning studies more commonly optimize location, capacity, cost, losses, voltage, or renewable integration through power-flow and mathematical-programming formulations [31-42]. Stochastic BESS expansion planning has explicitly represented uncertainty in demand,

[DOI: 10.71885/ijorlu-2026-4-743]

PV generation, workload, and outages [44]. This literature distinction is important: the present paper uses deterministic AC simulation followed by DEA benchmarking, whereas stochastic planning embeds uncertainty directly inside the investment model.

The economic literature emphasizes that battery value is service- and location-dependent. Power-related expenditure is mainly associated with converters, interconnection equipment, and rated power, whereas energy-related expenditure grows with storage capacity and duration [33,45,46]. Economic benefits may arise from avoided capacity expansion, reduced demand charges, lower losses, renewable-curtailment reduction, congestion relief, ancillary services, and energy arbitrage. However, combining all benefits and technical penalties in one monetary objective requires tariffs, shadow prices, degradation assumptions, and reliability values that are often uncertain or jurisdiction-specific. Multi-objective optimization avoids some monetization but still requires a rule for selecting one point from the Pareto frontier. DEA offers a complementary perspective by asking which observed alternative delivers the most favorable bundle of services and technical outcomes per unit of investment without imposing ex ante criterion weights.

3 Battery-alternative formulation and simulation framework

This section explains how a physical battery option is converted into a comparable decision-making unit. Each alternative is defined by location, rated power, and storage duration; its operating schedule is then mapped to the network and evaluated through hourly AC load flow.

Using a common schedule for alternatives with the same power and duration is a controlled experimental design. It isolates the electrical effect of location by preventing simultaneous changes in dispatch from obscuring spatial differences. The schedule is not claimed to be optimal for every bus. After screening, the selected location should be redispatched with explicit voltage, current, and state-of-charge constraints.

Separating power- and energy-related costs is essential for sensitivity analysis. Power cost is associated with conversion and interconnection equipment, while energy cost grows with cell and storage capacity. A single aggregated coefficient cannot reveal how declining cell prices alter the competitiveness of longer-duration batteries.

The radial network is represented by bus set N and branch set L. Hourly bus demand is obtained by scaling peak active and reactive loads, while PV output is derived from installed capacity and a normalized solar profile. Net active demand equals load minus PV plus battery charging minus discharging. The battery inverter is operated at unity power factor so that active-power siting effects are separated from reactive-voltage control. This is an experimental design choice, not a claim that modern inverters cannot provide reactive support. The feeder topology and base parameters follow the classical radial-distribution benchmark introduced by Baran and Wu [47,48], with the benchmark system cross-checked against MATPOWER data and the enhanced IEEE 33-bus formulation reported in later work [49,50].

For each alternative, rated power P_B and duration H define energy capacity E_B=P_BH. State-of-charge dynamics include charging and discharging efficiency, 10-90% energy bounds, and equal initial and terminal energy. The closed-cycle condition prevents artificial peak reduction by consuming initial energy without restoring it. Dispatch minimizes the maximum daily net demand with a small throughput penalty; charging is allowed during PV-production hours and discharging during the evening peak.

The dispatch objective is lexicographic in interpretation: the primary term minimizes the daily maximum net import, z, subject to Pnet(t) <= z for every hour. A strictly secondary

[DOI: 10.71885/ijorlu-2026-4-743]

throughput term, εΣt[Pch(t)+Pdis(t)], is included only to break ties among schedules that achieve the same peak. The coefficient ε is chosen sufficiently small that it cannot trade a measurable increase in peak import for a reduction in cycling. Its role is therefore to suppress unnecessary simultaneous or compensating charge-discharge throughput, not to represent battery degradation cost or an energy-price signal.

Candidate buses 6, 14, 18, 22, 25, 30, and 33 cover near-substation, intermediate, terminal, and lateral locations. Power ratings of 250, 500, and 750 kW are combined with durations of 2, 4, and 6 hours, yielding 63 alternatives. Duration is explicitly modeled because a two-hour battery is power-oriented whereas a six-hour device is energy-oriented.

Capital cost is decomposed into power-related and energy-related components: C=P_B(c_P+c_EH). This distinction is crucial because uniform cost scaling leaves DEA rankings unchanged, while relative changes in $/kW and $/kWh can shift preference between short- and long-duration systems. Dispatch is first determined for each power-duration pair and then placed at each candidate bus. This two-layer design isolates location effects; simultaneous co-optimization is identified as a subsequent implementation step. The component-based cost interpretation is consistent with recent utility-scale battery cost projections [45] and with broader life-cycle storage-cost analyses [33, 46].

$$P^{\text{net}}_{i,t,k} = P^{L}_{i,t} - P^{PV}_{i,t} + P^{ch}_{i,t,k} - P^{dis}_{i,t,k} \tag{1}$$

$$E^{B}_{k} = P^{B}_{k} H_{k} \tag{2}$$

$$e_{t+1,k} = e_{t,k} + \eta_c P^{ch}_{t,k}\Delta t - \frac{P^{dis}_{t,k}}{\eta_d}\Delta t \tag{3}$$

$$0.1E^{B}_{k} \le e_{t,k} \le 0.9E^{B}_{k}, \qquad e_{0,k} = e_{24,k} \tag{4}$$

$$C_k = P^{B}_{k}(c_P + c_E H_k) \tag{5}$$

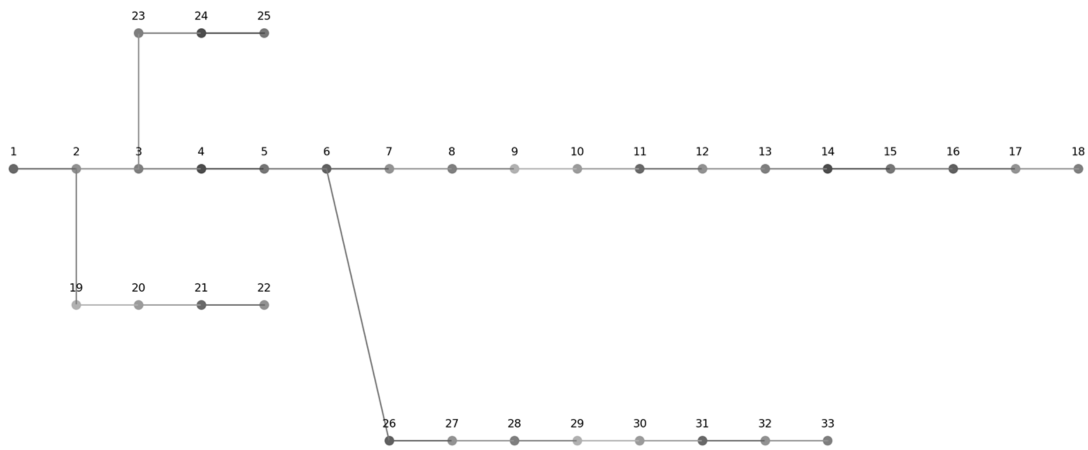

Fig. 1 IEEE 33-bus system and candidate battery buses

Figure 1 shows the radial topology and the distribution of candidate buses. Buses 6 and 14 represent intermediate main-feeder locations, buses 18 and 33 are terminal locations, and buses 22, 25, and 30 cover laterals with substantial load or PV. This spatial spread exposes the effect of electrical distance and the number of relieved upstream branches. Terminal discharge

[DOI: 10.71885/ijorlu-2026-4-743]

can reduce current over a longer path, but terminal charging is also more likely to aggravate voltage drop.

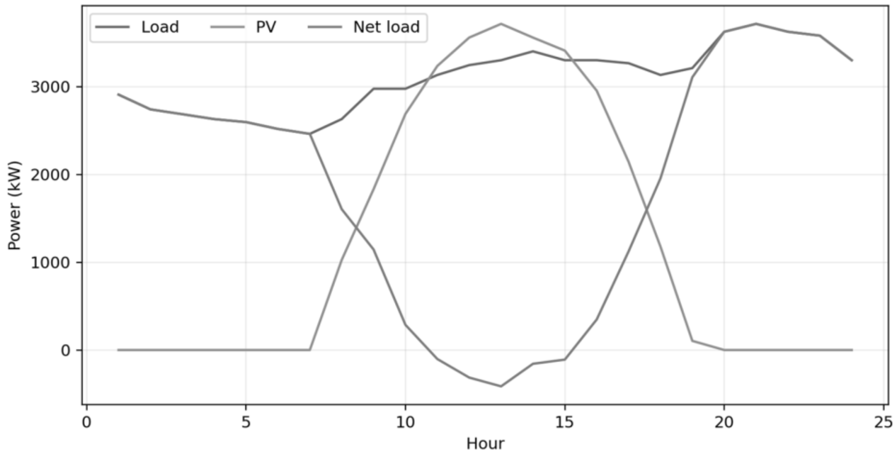

Fig. 2 Load and photovoltaic profiles

Figure 2 reveals the temporal mismatch between demand and PV production. Solar output peaks around midday, whereas the feeder peak occurs in the evening. PV alone therefore does not reduce the evening peak, and the battery creates value by transferring energy between the two periods. The width of the evening peak also explains why longer duration is useful up to a point, after which energy-capacity cost can increase faster than the additional service.

4 Technical-index calculation through AC load flow

This section derives the technical indicators directly from network physics. Because battery location affects branch current, losses, and bus voltage nonlinearly, nameplate ratings or an aggregate power balance are insufficient for comparing alternatives.

The backward-forward sweep is well suited to radial feeders because it uses the parent-child topology explicitly. The backward pass accumulates downstream currents, and the forward pass updates voltage drops from the slack bus to terminal buses. Repeating the two passes captures the dependence between current and voltage for constant-power loads. A strict convergence tolerance is used so that DEA rank differences are not contaminated by load-flow errors.

The four DEA indicators represent distinct services and consequences. Peak reduction measures upstream capacity relief, discharged energy measures service duration, electrical loss measures the physical cost of current flow, and VDI summarizes the overall voltage profile. Minimum voltage is reported in addition to VDI because an aggregate index can conceal a short local voltage violation.

Hourly AC power flow is solved by the backward-forward sweep algorithm. Bus injection currents are computed from complex power and voltage, branch currents are accumulated from terminal buses toward the slack bus, and downstream voltages are updated in the forward sweep. Iteration continues until the maximum voltage change falls below 10^-10. An AC model is retained because battery location changes current magnitude, resistive losses, and voltage nonlinearly.

Hourly branch losses are calculated as R|I|^2 and summed over 24 hours. Voltage deviation is the spatiotemporal sum of the absolute distance of bus-voltage magnitudes from

[DOI: 10.71885/ijorlu-2026-4-743]

one per unit. Minimum and maximum voltages are also recorded because two alternatives may have similar aggregate deviation but different extreme violations.

Peak reduction is measured at the slack bus as the difference between the no-battery and alternative-specific maximum active-power import. This definition captures the effect of network losses on upstream peak. Discharged energy is included separately so that short high-power support can be distinguished from sustained energy service.

Implementation was validated on the classical IEEE 33-bus peak case. Active loss was 202.677 kW, minimum voltage was 0.91309 p.u. at bus 18, and slack import was 3917.677 kW. The power balance equals 3715 kW of load plus losses. This validation is essential because a per-unit conversion error can generate numerically precise but physically meaningless DEA rankings.

$$I_{i,t,k}^{(q)} = \left(\frac{P_{i,t,k}^{\text{net}} + jQ_{i,t,k}^{\text{net}}}{V_{i,t,k}^{(q)}} \right)^* \tag{6}$$

$$I_{ij,t,k} = I_{j,t,k} + \sum_{m \in \mathcal{C}(j)} I_{jm,t,k} \tag{7}$$

$$V_{j,t,k} = V_{i,t,k} - Z_{ij} I_{ij,t,k} \tag{8}$$

$$E_k^{\text{loss}} = \sum_{t=1}^{24} \sum_{(i,j) \in \mathcal{L}} R_{ij} |I_{ij,t,k}|^2 \Delta t \tag{9}$$

$$VDI_k = \sum_{t=1}^{24} \sum_{i=2}^{33} |1 - |V_{i,t,k}|| \Delta t \tag{10}$$

$$\Delta P_k^{\text{peak}} = P_0^{\text{peak}} - P_k^{\text{peak}} \tag{11}$$

$$E_k^{\text{shift}} = \sum_{t=1}^{24} P_{t,k}^{dis} \Delta t \tag{12}$$

5 Undesirable-output SBM-DEA and the rationale for its selection

This section translates the technical and economic indicators into an efficiency model. SBM is selected because its assumptions match the observed response of battery alternatives; it is not adopted merely because it is common in DEA applications.

Radial CCR or BCC models require proportional changes in all inputs or outputs. Battery alternatives do not exhibit such behavior: cost excess, peak-output shortfall, and loss reduction can differ substantially. Non-radial SBM measures each slack separately and therefore avoids classifying a unit as strongly efficient when residual input or output slacks remain after radial projection.

The second reason is the treatment of adverse consequences. Loss and voltage deviation are neither consumed resources nor desirable products. The undesirable-output SBM reduces them directly within the production technology and avoids reciprocal or translation transformations that can alter relative distances and the frontier. The third reason is unit invariance, which allows dollars, kW, kWh, and p.u.-h to coexist without arbitrary normalization.

The SBM score must be interpreted together with slacks and raw indicators. A score of one means that no observed reference combination dominates the alternative by using less cost, producing more desirable services, and generating fewer undesirable consequences. It

[DOI: 10.71885/ijorlu-2026-4-743]

does not establish absolute optimality or network feasibility, because DEA constructs a relative frontier and physical constraints outside the selected variables must be checked separately.

Each battery alternative is a DMU. Investment cost is the sole input; peak reduction and discharged energy are desirable outputs; daily energy loss and voltage deviation are undesirable outputs. SBM is not chosen by convention. First, CCR and BCC radial models impose proportional improvement, although a battery alternative may have a large cost excess, a moderate peak-output shortfall, and only a small loss gap. Second, SBM incorporates every slack in the score and avoids declaring a unit efficient after a radial movement while nonzero slacks remain. Third, its undesirable-output form reduces losses and voltage deviation directly without reciprocals or arbitrary translations. Fourth, unit invariance is indispensable when dollars, kW, kWh, and p.u.-h coexist.

The observed vector of DMUo is compared with a reference combination and input, good-output, and bad-output slacks. In the undesirable-output constraint, the benchmark equals the observed bad output minus its slack; a positive slack therefore has a direct interpretation as feasible loss or voltage-deviation reduction. Strong SBM efficiency requires a score of one and zero slacks.

The main model assumes constant returns to scale because battery size is itself a planning decision and an oversized system should be penalized for inappropriate scale. A VRS version is solved as sensitivity analysis to separate scale from local technical performance. Efficient units are ranked with Super-SBM by excluding the evaluated unit from the reference technology. Because super-efficiency differences can be very small, raw engineering indicators and sensitivity results are reported alongside the score.

$$\rho_o = \frac{1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1 + \frac{1}{s_g + s_b}\left(\sum_{r=1}^{s_g}\frac{s_r^g}{y_{ro}^g} + \sum_{\ell=1}^{s_b}\frac{s_\ell^b}{b_{\ell o}}\right)} \quad (13)$$

$$x_o = X\lambda + s^-, \qquad y_o^g = Y^g\lambda - s^g, \qquad b_o = B\lambda + s^b \quad (14)$$

$$\lambda \geq 0, \qquad s^- \geq 0, \qquad s^g \geq 0, \qquad s^b \geq 0 \quad (15)$$

Table 1 DEA variables, roles, and preferred directions

Variable	Symbol	Role	Preferred direction
Capital cost	C	Input	Decrease
Peak reduction	PR	Desirable output	Increase
Delivered energy	Edis	Desirable output	Increase
Daily energy loss	Eloss	Undesirable output	Decrease
Voltage deviation index	VDI	Undesirable output	Decrease

6 Case study, results, ranking, and engineering interpretation

This section presents the case data, load-flow outputs, DEA ranking, and engineering interpretation as one connected analysis. The objective is not merely to name the highest-ranked alternative, but to explain why alternatives that excel in one technical index can receive a lower overall efficiency rank.

[DOI: 10.71885/ijorlu-2026-4-743]

The leading alternative should not be interpreted as the scenario with the lowest value of every adverse indicator. B14-P750-D4 does not minimize loss or VDI; it lies at the frontier because its investment produces a comparatively strong joint bundle of peak relief and delivered energy while keeping loss and voltage-deviation consequences within the performance envelope formed by the observed alternatives. Economically, the result indicates a favorable marginal use of capital rather than the highest absolute technical benefit. From a planning perspective, this distinction is important: a larger battery may produce more physical service but still be inefficient if the incremental service is smaller than the incremental power- and energy-related expenditure.

The narrow super-efficiency gaps among the first three alternatives imply that the ranking is not a universal prescription. Bus 14 provides a strong compromise between capital cost and upstream peak reduction. Bus 22 offers a more favorable voltage outcome and becomes more attractive when energy duration is valued more highly. Bus 30 relieves current in several upstream sections and therefore produces strong effective loss and peak benefits. These differences originate in feeder topology, load distribution, local PV production, and the timing of battery operation. Consequently, the final investment decision should combine the DEA frontier with tariff structure, voltage limits, thermal headroom, land availability, interconnection cost, and the operator's intended service portfolio.

The IEEE 33-bus feeder contains 33 buses and 32 radial branches at 12.66 kV with 3.715 MW and 2.3 MVAr peak demand. Base-case PV capacity equals 100% of peak active load and is distributed among buses 18, 22, 25, and 33. The no-battery day produces 2530.04 kWh of loss, a 3917.68-kW upstream peak, a voltage-deviation index of 24.0051 p.u.-h, and a minimum voltage of 0.91309 p.u. These values establish the physical baseline against which the storage alternatives are assessed. The relatively low terminal voltage confirms that the feeder is electrically sensitive, so the location and charging period of a large battery can be as important as its rated power.

Eleven of the 63 alternatives are CRS-SBM efficient. B14-P750-D4 ranks first with a $1.002 million cost, 667.27-kW peak reduction, 2212.69-kWh discharged energy, 2476.26-kWh loss, and a 1.000930 super-efficiency score. Relative to the no-battery case, it reduces the upstream peak by 17.03% and daily losses by 2.13%, while aggregate voltage deviation increases slightly. The economic implication is that the four-hour design uses its energy capacity more intensively during the relevant peak window than a longer-duration design whose additional cells may remain underutilized for the selected service. The engineering implication is that the intermediate location provides meaningful upstream-current relief without imposing the severe charging-hour voltage penalty observed at the weakest terminal buses.

B22-P750-D6 and B30-P750-D6 rank second and third, but they create value through different mechanisms. The bus-22 alternative benefits from proximity to a lateral containing substantial load and PV, which improves local energy balancing and voltage performance. The bus-30 alternative is electrically deeper in the network and its discharge reduces current over a longer common upstream path; because branch loss varies approximately with the square of current, modest current relief across several resistive sections can generate a material daily loss reduction. Economically, their six-hour duration increases energy-related cost. They remain competitive only because the additional duration produces sufficient discharged energy and network service to compensate for that cost. This explains why a long-duration battery is not automatically efficient and why its attractiveness changes when the energy-cost component is varied.

[DOI: 10.71885/ijorlu-2026-4-743]

The minimum-loss alternative, B18-P500-D6, records 2428.53 kWh but ranks 35th. It is technically attractive from a loss perspective, yet its $0.909 million investment and 549.26-kW peak reduction are dominated in the joint input-output space by alternatives that deliver a more balanced service bundle. The best-VDI alternative, B22-P500-D6, ranks 53rd for the same reason. These cases demonstrate the economic meaning of the DEA assessment: minimizing a single engineering index can allocate additional capital to a benefit that has already entered a region of diminishing returns. Conversely, B18-P250-D2 can be efficient despite modest absolute benefits because its lower investment yields a strong benefit-to-cost pattern. The frontier therefore contains both high-capacity strategic assets and smaller, capital-efficient options.

Independent benchmark comparison. The DEA result was also checked against direct single-criterion optimization over the same finite set of 63 simulated alternatives and against the no-battery operating case. Direct loss minimization selects B18-P500-D6 (2428.53 kWh/day), whereas direct voltage-deviation minimization selects B22-P500-D6; these alternatives rank 35th and 53rd, respectively, in the joint DEA assessment. Conversely, the DEA-leading B14-P750-D4 reduces the upstream peak by 667.27 kW and daily losses relative to the no-battery case, but it is not the minimum-loss or minimum-VDI solution. This cross-check is important: the proposed DEA layer does not reproduce a hidden single-objective optimizer; it identifies relative capital-use efficiency when several desirable and undesirable outcomes are considered simultaneously.

The 0.8965-p.u. minimum voltage obtained for B18-P750-D6 is a critical feasibility warning. During discharge, an end-of-feeder battery reduces upstream current and can support voltage; during charging, the same unit becomes a large concentrated load at the electrically weakest location. If charging occurs after PV output begins to decline, the branch current and voltage drop increase simultaneously. The peak-shaving dispatch used in this study does not include a local voltage constraint, so it can schedule an economically useful energy transfer that is unacceptable from an operating standpoint. This result confirms that DEA is a screening and benchmarking tool: any alternative violating statutory voltage or thermal limits must be excluded, resized, relocated, or redispatched before investment approval.

DEA efficiency and network admissibility are reported separately. In the simulated data, the B18-P750-D4 and B18-P750-D6 alternatives reach a minimum voltage of approximately 0.8965 p.u. and are therefore flagged as voltage-inadmissible under a 0.90-p.u. screening bound used only for this case-study check. They must not be promoted for implementation on the basis of DEA rank. The classical IEEE 33-bus data set used here does not provide a complete, study-specific set of branch ampacity ratings; consequently, thermal feasibility is not claimed. A practical planning study must add utility-specific line/transformer ratings and remove or redispatch any alternative that violates those limits before DEA-based economic screening is used for investment selection.

The cost sensitivity results have a direct economic interpretation. Multiplying all battery costs by the same positive factor does not change DEA efficiency because the common rescaling leaves relative production possibilities unchanged. In contrast, changing the ratio between power-related and energy-related costs changes the economic structure of the alternatives. A 30% reduction in the energy component moves B22-P750-D6 to first place because the incremental cost of six-hour storage falls while its delivered energy and network benefits remain unchanged. This result is particularly relevant when technology learning reduces cell cost faster than converter and interconnection cost. The high rank correlations, all above 0.97 for the price cases, show that the frontier is stable even though the preferred duration may change.

[DOI: 10.71885/ijorlu-2026-4-743]

PV and load sensitivities reveal the power-system mechanisms behind rank changes. At 50% PV penetration, B6-P750-D6 leads because surplus midday energy is limited and the dominant service is relief of the main feeder and evening peak. At 150% PV penetration, B18-P750-D6 becomes more valuable because storage near the feeder end absorbs local surplus, reduces reverse transfer through upstream branches, and releases that energy during the evening peak. Under 80% loading, B30-P750-D6 leads because deeper placement provides effective loss and peak relief without the same degree of charging stress. At 120% loading, B14-P750-D4 remains first, indicating that an intermediate location offers a robust compromise when both branch currents and voltage sensitivity increase. The VRS model identifies 19 efficient units compared with 11 under CRS, and the 0.8414 rank correlation confirms that scale choice materially affects investment interpretation. Taken together, the results show that location determines the electrical leverage of each kilowatt, duration determines how long the service can be sustained, and the relative cost of power and energy determines whether that technical service is economically justified.

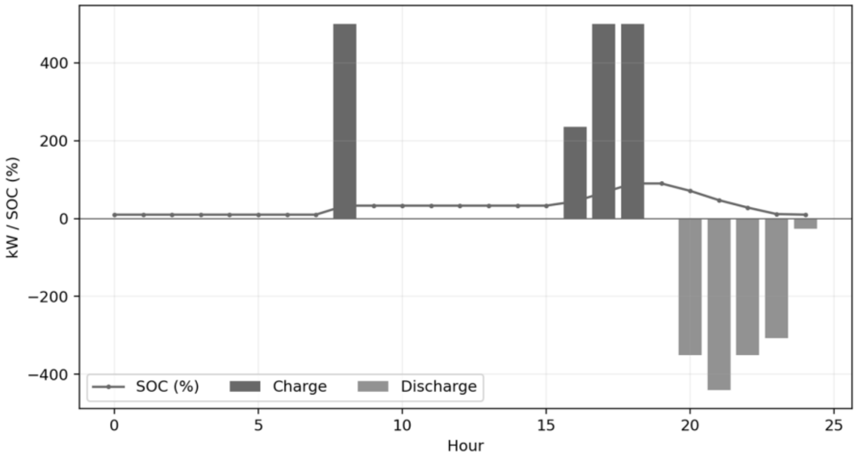

Fig. 3 Representative battery charging and discharging schedule

Figure 3 shows that charging is concentrated in the PV-production interval and discharging is shifted to the evening demand peak. The flat portions near rated charge or discharge power indicate that converter capacity is binding, whereas the termination of discharge reflects the available energy capacity and minimum state-of-charge limit. Economically, operation near rated power increases utilization of the power-related investment; however, extending duration beyond the peak window adds energy capacity that may not be fully remunerated by peak-shaving service alone. Electrically, the same schedule has different consequences at different buses. At an intermediate bus, charging current is distributed over a shorter weak path, while at a terminal bus it flows through nearly the entire feeder and can depress voltage. The figure therefore explains why identical battery ratings and schedules obtain different DEA scores after location-specific load flow.

Table 2 Top-ranked battery installation scenarios in the base case

Rank	Scenario	Cost (thousand USD)	Peak reduction (kW)	Energy (kWh)	Loss (kWh)	VDI	Score
1	B14-P750-D4	1002.0	667.27	2212.69	2476.26	24.2002	1.000930

[DOI: 10.71885/ijorlu-2026-4-743]

Rank	Scenario	Cost (thousand USD)	Peak reduction (kW)	Energy (kWh)	Loss (kWh)	VDI	Score
2	B22-P750-D6	1363.5	749.16	2917.98	2522.32	23.9561	1.000919
3	B30-P750-D6	1363.5	813.95	2917.98	2445.35	24.1542	1.000642
4	B6-P750-D6	1363.5	800.25	2917.98	2443.76	24.1038	1.000448
5	B14-P500-D4	668.0	509.31	1475.13	2456.08	24.1046	1.000403
6	B14-P750-D6	1363.5	815.32	2917.98	2451.32	24.2508	1.000235
7	B18-P250-D2	213.5	215.78	368.78	2504.56	24.0261	1.000193
8	B18-P500-D4	668.0	508.49	1475.13	2442.84	24.1036	1.000118
9	B22-P750-D4	1002.0	609.79	2212.69	2525.80	23.9650	1.000088
10	B6-P750-D4	1002.0	651.17	2212.69	2470.04	24.0880	1.000038

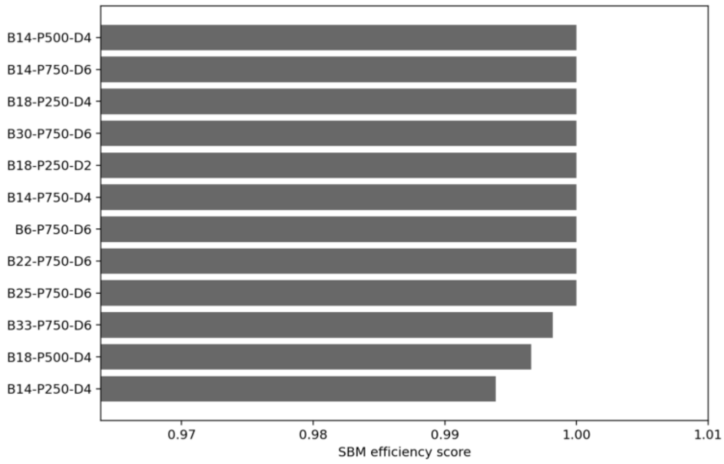

Fig. 4 Comparison of leading alternatives by SBM score

Figure 4 shows that the leading SBM and super-efficiency values are tightly clustered. This pattern means that several alternatives form a technically and economically competitive frontier rather than one scenario dominating by a wide margin. The first-place alternative has the most balanced observed conversion of capital into peak reduction and delivered energy after accounting for loss and voltage deviation, but the small score differences make the ranking sensitive to legitimate planning priorities. A utility facing an imminent transformer-capacity constraint may favor the alternative with stronger peak relief, whereas a feeder with limited voltage margin may select a slightly lower-ranked option with better VDI and minimum voltage. The chart should therefore be read as a shortlist of efficient designs, not as a substitute for detailed project appraisal.

[DOI: 10.71885/ijorlu-2026-4-743]

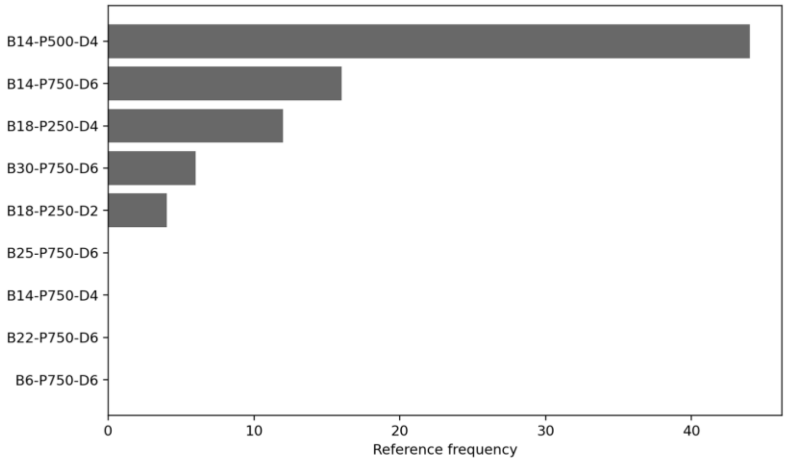

Fig. 5 Reference-set frequency of alternatives

Figure 5 reports reference-set frequency, which provides a different interpretation from super-efficiency. An alternative that appears frequently in reference sets supplies a recurring combination of cost, service, and technical outcomes that inefficient DMUs can emulate. High frequency therefore identifies a practically influential benchmark, even when that unit is not first in the super-efficiency ranking. From an engineering viewpoint, such a scenario represents a configuration whose location and size repeatedly provide attainable improvement targets. From an economic viewpoint, it indicates a capital-service structure that remains relevant across several inefficient scales. The measure is useful for portfolio design because it distinguishes a robust benchmark from a narrowly specialized frontier point.

7 Sensitivity analysis

The sensitivity analysis examines whether rankings are an artifact of the base assumptions or whether the main performance pattern persists under alternative economic and operating conditions. Battery cost components, PV penetration, loading, and returns to scale are varied separately so that each source of rank change remains interpretable.

High rank correlations indicate that the overall frontier is stable, while changes in the leading alternative remain economically and physically interpretable. Higher PV penetration increases the marginal value of local energy absorption and reverse-flow mitigation at terminal buses. Higher loading increases the value of reducing current in common upstream branches because both voltage drop and resistive loss become more sensitive to current. A lower energy-component cost improves long-duration batteries because a greater share of their capital is associated with cells rather than converters. These rank changes therefore reflect altered service value and network stress, not numerical instability.

Sensitivity analysis varies power- and energy-related battery costs independently by ±30%, PV penetration across 50%, 100%, and 150%, loading across 80%, 100%, and 120%, and returns to scale from CRS to VRS. Stability is evaluated through Spearman rank correlation and changes in the leading alternatives.

The present paper does not formulate load, PV generation, or market energy price as random variables and therefore does not claim stochastic or robust optimality. Load and PV uncertainty are addressed only as deterministic stress scenarios (80%-120% loading and 50%-150% PV penetration), while battery power- and energy-related capital-cost coefficients are

[DOI: 10.71885/ijorlu-2026-4-743]

perturbed by ±30%. Energy-market price is not an input to the dispatch model because the operating objective is peak shaving rather than energy arbitrage; its uncertainty is therefore outside the scope of the reported numerical results. This distinction has been made explicit to avoid presenting scenario sensitivity as probabilistic uncertainty analysis. Stochastic load/PV trajectories and energy-price processes are reserved for future work. Explicit stochastic BESS-planning formulations provide an appropriate reference point for such an extension [44].

Because DEA frontiers depend on the observed comparison set, an additional leave-one-frontier-alternative-out diagnostic was performed on the 63-alternative data matrix. Each frontier alternative was removed in turn and the remaining alternatives were re-evaluated. Spearman correlations between the original and perturbed SBM scores for the common alternatives remained approximately 0.998-1.000, indicating that the broad efficiency ordering is highly stable to deletion of a single frontier option. The test nevertheless reinforces the interpretation that very small super-efficiency differences among the leading alternatives should be treated as a competitive frontier rather than as an absolute ordering. This diagnostic follows the reference-set sensitivity logic developed in the DEA literature [51, 52].

The rankings are robust to moderate parameter changes, but the winner can change when the economic or electrical environment changes materially. B14-P750-D4, B22-P750-D6, and B30-P750-D6 form a robust core of leading alternatives. Bus 14 is preferable when investment efficiency and peak reduction are balanced; bus 22 becomes more attractive when voltage quality, local PV balancing, and sustained energy delivery receive greater emphasis; and bus 30 is competitive when relieving downstream current and reducing losses over a long upstream path are central. For investment planning, the sensitivity results imply that a single deterministic rank should be accompanied by scenario-specific rankings tied to expected PV growth, demand evolution, and battery cost trajectories.

Table 3 Sensitivity of the three leading scenarios to price, PV penetration, and load level

Case	First-ranked scenario	Second-ranked scenario	Third-ranked scenario	Correlation with base case
Energy cost-30%	B22-P750-D6	B14-P750-D4	B30-P750-D6	0.9910
Base case	B14-P750-D4	B22-P750-D6	B30-P750-D6	1.0000
Energy cost +30%	B14-P750-D4	B22-P750-D6	B30-P750-D6	0.9868
PV=50%	B6-P750-D6	B30-P750-D6	B14-P500-D4	0.9720
PV=150%	B18-P750-D6	B14-P750-D6	B14-P750-D4	0.9454
Load=80%	B30-P750-D6	B14-P500-D4	B18-P250-D4	0.9542
Load=120%	B14-P750-D4	B14-P750-D6	B22-P750-D6	0.9581

8 Conclusions

This section summarizes the findings in relation to the research question and defines the scope of validity. Conclusions are restricted to the computations performed in this study, and results from one benchmark feeder are not generalized to all distribution networks.

This paper presented an integrated load-flow and undesirable-output SBM-DEA framework for battery siting and sizing alternatives. The framework compares heterogeneous units without arbitrary objective weights and directly treats losses and voltage deviation as bad outputs. The minimum-loss, best-voltage, and maximum-peak-reduction alternatives are not necessarily identical, and efficiency emerges from the trade-off between benefit and capital.

[DOI: 10.71885/ijorlu-2026-4-743]

SBM is justified by non-radial responses, explicit slack treatment, unit invariance, and direct undesirable-output modeling. On the IEEE 33-bus system, B14-P750-D4 is the base-case leader, although the best alternative changes with PV penetration and cost structure. Peak-only dispatch can also create end-of-feeder voltage violations.

Limitations include a representative deterministic day, balanced modeling, fixed battery efficiency, omission of degradation, absence of probabilistic load/PV and energy-price uncertainty, incomplete thermal-rating data for a formal ampacity screen, and sequential rather than simultaneous dispatch-location optimization. The reported load, PV, and capital-cost variations are stress tests rather than stochastic uncertainty models. Future work should include unbalanced three-phase power flow, probabilistic or distributionally robust scenarios, energy-price uncertainty, degradation cost, inverter reactive control, utility-specific thermal constraints, and network or dynamic DEA. The most defensible use of the proposed framework is screening and explaining alternatives before detailed stochastic or robust investment optimization.

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[DOI: 10.71885/ijorlu-2026-4-743]