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The selection of a single and acceptable target in Data Envelopment Analysis

B. Daneshyan*, R. Maleki Vishkaei, B. Maleki Vishkaei

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Abstract There are several systems in real situations that only accept a single and acceptable benchmark for efficiency improvement. For instance, when we are going to select a target for a country, the target country should be as close as the view of culture, economics and politics to the inefficient country. Otherwise, it would not be acceptable. Therefore, an inefficient country for several reasons can't accept any efficient country for benchmarking. In this Paper we focus on this subject and show it in a real practical case.

Keywords Target Selection, Single and Acceptable Benchmark, DEA.

1 Introduction

Farrell [1] studied a model for evaluating the efficiency considering multi inputs and a unique output. After two decades or so Charnes [2] developed this technique to multi outputs via introducing the CCR model and called it Data Envelopment Analysis (DEA). Benker [3] developed BCC model for evaluating the decision making units. According to these models decision making units are classified into efficient and inefficient groups. In CCR and BCC models the evaluating set is compared with a virtual unit which is a combination of efficient units. The virtual unit may not be acceptable practically. To solve this problem, FDH models are presented by Deprins, Simar, Tulkenes [4]. These models are according to convexity principle for creating PPS. In FDH models there is no need to solve a linear programming problem and the solution will be easily found via Paired comparison. In next sections first of all two simple examples are studied; the first example includes two inputs and one output and, in the second example two inputs and two outputs are considered. The SSAT algorithm is used for solving these examples. Moreover, a practical example is presented for the proposed algorithm. The last section gives a brief conclusion.

* **Corresponding Author.** (✉)
E-mail:

B. Daneshyan, R. Maleki Vishkaei
Department of Mathematic, Islamic Azad University,Lahijan Branch, Lahijan, Iran

B. Maleki Vishkaei
Department of Industrial Engineering, Islamic Azad University, Qazvin Branch, Qazvin, Iran

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2 SSAT algorithm for a single and acceptable target

Data Envelopment Analysis provides a set of beneficial linear programming procedure for evaluating the efficiency of the decision making units. To increase the efficiency of an inefficient unit, a target set is considered for it. This set is a combination of efficient units. Although this kind of target set can be beneficial, there are examples that because of different results can't use this method, for instance, consider few jet plane engines are comparing with each other. Obviously, for an inefficient engine a target set which is a combination of jet engines can't be considered (it's not possible technically). Before, Deprins, Simar, Tulkenes [4] studied the problem using FHD models. In this model a single target is allocated to an inefficient unit, and the problem of multiple targets is vanished.

This paper is organized to prove the acceptability of the target is important for the inefficient unit too. Acceptability means an inefficient unit should follow a single target which is acceptable in reality.

As an example, when we are evaluating the countries, a combination of few countries can't be chosen as a pattern for a progressing country (inefficient unit). Although using FHD model leads to select a single country as the target, the inefficient country may not follow the target country because of basic cultural, political or race differences. Therefore, all the single efficient countries that are selected for an inefficient country would not be acceptable.

SSAT algorithm is presented to make sure the chosen target for inefficient unit is single and acceptable and the inefficient unit try to achieve it using a defined method to clarify the problem two simple examples are studied.

Example 1. Table (1) shows the data of decision making units.

Table 1 Data of decision making units

G	F	E	D	C	B	A	DMU
3	10	2	4	8	7	4	X1
7	1	4	2	1	3	3	X2
1	1	1	1	1	1	1	Y

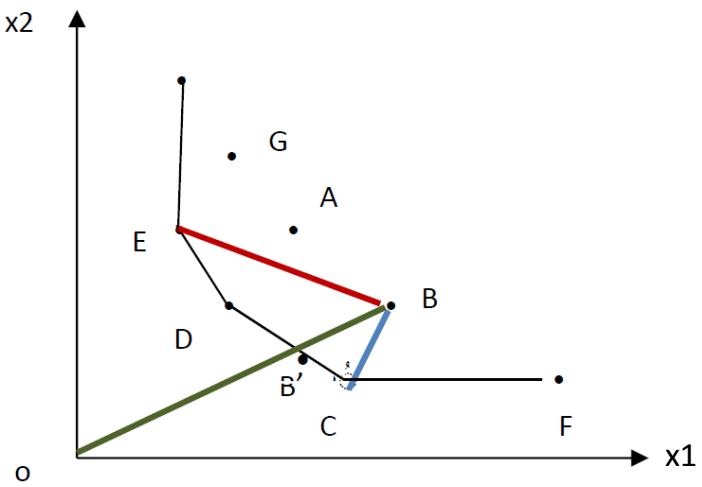

Fig. 1

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The results of solving the example by CCR-I are shown in table (2).

Table 2 Result of CCR-I for the example

S^+	S_i^-	S_i^-	source	efficiency	DMU
0	0	0	D,E	0.8571	A
0	0	0	C,D	0.6316	B
0	0	0	C	1	C
0	0	0	D	1	D
0	0	0	E	1	E
0	0	2	C	1	F
0	0.6667	0	E	0.6667	G

The target B' which includes efficient units C,D is considered for unit B. $\lambda_C = \circ.1053$ and $\lambda_D = \circ.8947$ are the coefficient amounts of units C and D for virtual unit B'.

If decision making unit B does not choose B' as the pattern like what occurred in jet plane engines or country examples, a single target must be used. B can choose one of the units C or D which are the components of virtual unit B' as the single target or even can choose efficient unit E as its pattern. After that inefficiency unit B has selected its target, it can achieve to its target using SSAT algorithm via Path made of a convex combination.

Consider that among efficient units C, D, E inefficiency unit B selects unit E as the single and acceptable target. According to figure (1) unit B could select one of the paths BE, BD or BC to increase its efficiency. The recommended path for DEA is BB'. Point that target E is not a member of virtual unit B'.

Example 2. Data for 12 hospitals are brought in table (3).

Table 3 Data of hospital example

Hospital	I_1	I_2	O_1	O_2
A	20	151	100	90
B	19	131	150	50
C	25	160	160	55
D	27	168	180	72
E	22	158	94	66
F	55	255	230	90
G	33	235	220	88
H	31	206	152	80
I	30	244	190	100
J	50	268	250	100
K	53	306	260	147
L	38	284	250	120

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I_1 and I_2 show the number of nurses and doctors respectively. O_1 and O_2 are the outputs and show the number of patient who are not admitted and admitted respectively. The result of running BCC-I model is shown in table (4).

Table 4 Results of BCC-I

Hospital	efficiency	source
A	1	A(1)
B	1	B(1)
C	0.895833	B(0.67),D(0.33)
D	1	D(1)
E	0.881818	A(0.4),B(0.6)
F	0.938936	D(0.29),J(0.71)
G	1	G(1)
H	0.798833	A(0.37),D(0.61),K(7.05E-03),L(1.64E-02)
I	0.989333	A(0.29),B(0.16),L(0.54667)
J	1	J(1)
K	1	K(1)
L	1	L(1)

If we need to increase the efficiency of hospital H more than 0.798823, its single target can be chosen from table (5).this table show units A, D, K, L with their coefficients.

Table 5

reference	coefficient (λ)
A	0.37
D	0.61
K	7.05E-03
L	1.64E-02

Consider hospital H chooses hospital D as its single and acceptable target. Point that single and acceptable target is chosen from components of multi target. After introducing SSAT algorithm, it will be clear how hospital H move toward its pattern.

3 SSAT algorithm

1. Chose the appropriate model for data.
2. Run the model and define the target units.
3. Define the inefficiency unit that you must to improve it.
4. For the inefficient unit chose a single and acceptable target from the list of efficient units.
5. Create the convex combination of the single and acceptable unit and the inefficient unit.
6. Via appropriate steps in convex combination define the table of efficiency improvement and the path for achieving the single and acceptable target.
7. Continue step 6 until inefficient unit achieve to its pattern.

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The action of SSAT algorithm for example 1 is shown in table (6).

$$\mathrm{B_i} = \lambda_i B + (1-\lambda_i)E \qquad \mathrm{i} = 1,2,...,\mathrm{n} \qquad \lambda_\mathrm{n} = \circ \tag{1}$$

Table 6

i	λ_i	Efficiency	I_1	I_2	o_1
1	1	0.631579	7	3	1
2	0.9	0.634921	6.5	3.1	1
3	0.8	0.652174	6	3.2	1
4	0.7	0.681818	5.5	3.3	1
5	0.6	0.714286	5	3.4	1
6	0.5	0.75	4.5	3.5	1
7	0.4	0.789474	4	3.6	1
8	0.3	0.833333	3.5	3.7	1
9	0.2	0.882353	3	3.8	1
10	0.1	0.9375	2.5	3.9	1
11	0	1	2	4	1

According to table (6) decision making unit B with efficiency 0.631579, $\lambda_1 = 1$ and length of steps 0.1 moves toward efficient unit E on the path BE. The input and output of unit B is changing during each step.
The action of SSAT algorithm for example 2 is shown in table (7).

$$\mathrm{H_i} = \lambda_i H + (1-\lambda_i)\mathrm{D} \qquad \mathrm{i} = 1,2,...,\mathrm{n} \qquad \lambda_\mathrm{n} = \circ \tag{2}$$

Table 7

λ	Efficiency	I_1	I_2	o_1	o_2
1	0.798	31	206	152	80
0.8	0.833	30.2	198.4	157.6	78/4
0.6	0.870	29.4	190.8	163.2	76/8
0.4	0.910	28.6	183.2	168.8	75/2
0.2	0.953	27.8	175.6	174.4	73/6
0	1	27	168	180	72

4 Using SSAT for evaluating universities

As the evaluation of universities has direct influences on industrial, cultural, agricultural and medicine organizations, improving their efficiency seems to be so essential. There are different universities which can be categorized to industrial, humanism and so on. Therefore, to improve a university quality in education, research and culture a multiple target is not suitable. SSAT can be a solution in this important case. Using data from Beasley etc. [3, 4] paper, a single and acceptable target is chosen for an inefficient university in next example. A part of running model BCC-I for 50 universities is shown in table (8).

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Table 8

universities	efficiency	University authorities
U11	0.830537	U1(0.68919),U45(0.31081)
U12	0.717748	U1(0.20908),U7(0.45674),U20(6.00E-02),U34(0.27422)
U13	0.849951	U1(0.64218),U25(5.21E-02),U49(0.30571)
U14	0.705328	U1(0.6459),U41(0.32356),U49(3.05E-02)
U15	0.67666	U1(0.69955),U7(4.47E-02),U35(0.25579)
U16	0.561452	U1(0.85357),U20(4.07E-02),U33(8.54E-02),U46(2.03E-02)
U17	0.555228	U1(0.81839),U6(1.02E-02),U49(0.17141)
U18	0.639125	U1(0.35442),U35(0.37096),U45(0.27463)
U19	1	U19(1)
U20	1	U20(1)
U21	0.706729	U1(0.7212),U6(0.10402),U20(8.00E-02),U34(9.48E-02),U46(1.36E-02)
U22	0.726497	U1(0.38431),U7(0.36124),U20(1.80E-02),U45(0.22277),U46(1.36E-02)
U23	0.560606	U1(0.74451),U34(0.19361),U35(6.10E-02),U45(8.97E-04)
U24	0.629432	U7(0.21495),U20(0.12907),U33(0.20154),U36(0.45444)
U25	1	U25(1)

Table (9) gives the list of 16 efficient universities.

Table 9

U1	U20	U34	U44
U6	U25	U35	U45
U7	U30	U36	U46
U19	U33	U41	U49

The goal is improving the efficiency of u23 (its current efficiency is 0.560606). The target universities which combine to make u23 are shown in table (10).

Table 10

Efficient unit	Coefficient
U1	0.74451
U34	0.19361
U35	6.10E-02
U45	8.97E-04

Manager of u23 after surveying found that u1 is the only proper university for improving the efficiency according to SSAT algorithms movement of u23 toward efficient unit u1 is shown in table (11).

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Table 11

λ	efficiency	I_1	I_2	o_1	o_2	o_3
1	0.560606	1115	450	148	4	32
0.9	0.582569	1056.3	411.4	147.7	3.6	31.4
0.8	0.607117	997.6	372.8	147.4	3.2	30.8
0.7	0.634735	938.9	334.2	147.1	2.8	30.2
0.6	0.666036	880.2	259.6	146.8	2.4	29.6
0.5	0.701811	821.5	257	146.5	2	29
0.4	0.743091	762.8	218.4	146.2	1.6	28.4
0.3	0.791255	704.1	179.8	145.9	1.2	27.8
0.2	0.836514	645.4	141.2	145.6	0.8	27.2
0.1	0.916495	586.7	102.6	145.3	0.4	26.6
0	1	528	64	145	0	26

Note that in each step the coefficient of efficient units which make the multiple targets is reducing and the weight of single target unit is increasing to its higher possible amount. More over the targets and efficiency of other units during these steps are solid.

5 Conclusions

By using a SSAT algorithm, an acceptable target will be chosen for the inefficient units and the path of moving inefficiency unit toward the single and acceptable target will be defined. During each step inputs and outputs of the efficient unit will be changed in this model inefficient unit achieve the efficient unit by defining a single and acceptable target and a proper path.

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