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Waste Collection Vehicle Routing Problem Considering Similarity Pattern of Trashcan

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Abstract Collection of waste is an important logistic activity within any city. In this study, a mathematical model is proposed in order to reduce the cost of waste collection. First a mixed-integer nonlinear programming model is provided including a waste collection routing problem, so that, there is a balance between the distance between trashcans, and the similarity of the trashcans in terms of the types of waste which is in the trashcans, in order to select the optimal route for each garbage transport vehicles. For this purpose, a similarity pattern is designed. Then, the model is solved by using LINGO 0.9 software and the output results of solved model were analyze.

Keywords Reverse Supply Chain, Recycling, Vehicle Routing Problem, Mathematical Modeling.

1 Introduction

We address a waste collection VRP with consideration of similarity of the trashcans in terms of the types of waste which is in the trashcans. The waste collection problem consists of routing vehicles to collect customers waste while minimizing travel cost. This problem is known as the Waste Collection Vehicle Routing Problem (WCVRP). WCVRP differs from the traditional VRP by that the waste collecting vehicles must empty their load at disposal sites. The vehicles must be empty when returning to the depot. The problem is illustrated in Fig. 1 for a one disposal site with a set of vehicles.

The structure of the present paper is the following: Section 2 discusses the literature which has previously studied WCVRP or other similar and relevant problems. The WCVRP is then formulated with consideration of similarity of the trashcans in terms of the types of waste which is in the trashcans and modeled in section 3. The subsequent section 4 discusses the results obtained from LINGO software. Finally, our concluding remarks are given in section 5.

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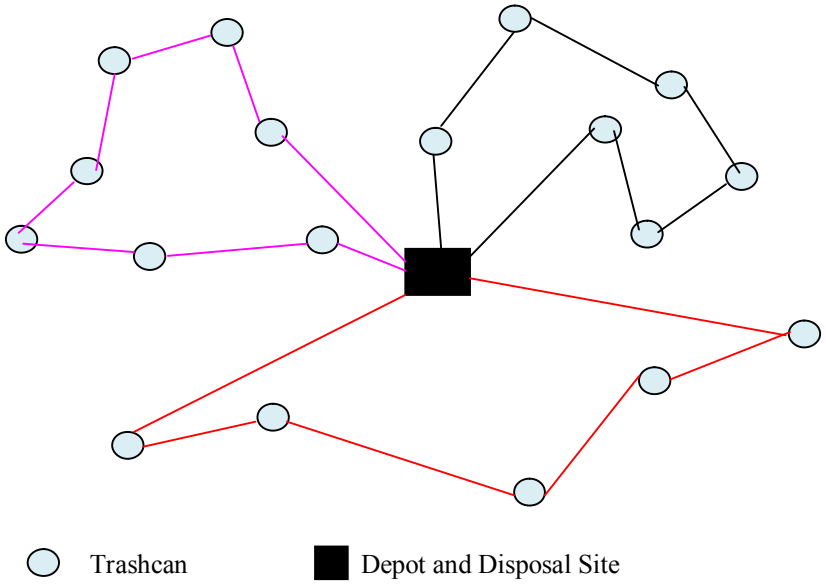

Fig. 1 A route sequence of some vehicle considering disposal operations with single disposal sites

2 Review of related literature

Taillard *et al.* [1] used a new edge exchange heuristic named CROSS and tabu search to improve the vehicle routing solutions. They restrict exchanging of customers between any two routes to subsets of consecutive customers. Constant time methods of evaluation and a feasibility test for an exchanged solution using an approximation function are also presented. To speed up computation, a decomposition and reconstruction approach is used. Taillard et al. [2] consider a vehicle routing problem with multiple use of vehicles in which multiple routes can be served by the same vehicle during a planning period and present a tabu search heuristic for the problem. The time windows of the stops and intermediate facilities such as landfills are not considered in their discussion.

Weigel and Cao [3] presented a case study of application of VRPTW algorithms for Sears home delivery problem and technician dispatching problem. They followed a cluster-first-route-second method and discuss three main routines: origin-destination (OD) matrix construction, route assignment, and route improvement routines. They applied a shortest-path algorithm to a geographic information system (GIS) to obtain OD matrix, i.e., travel time between any two stops. For the route assignment routine (clustering), an algorithm called multiple-insertion, which is similar to the parallel insertion algorithm of Potvin and Rousseau [4] is developed. As an objective function, the weighted combination of travel time, wait time, and time window violation is used. They propose an intra-route improvement algorithm and a neighborhood inter-route improvement algorithm that improves the solution quality by transferring and exchanging stops between two routes. In order to enhance the improvement performance, tabu search is applied to the improvement algorithms.

Tung and Pinnoi [5] modified Solomon's insertion algorithm and apply it to a waste collection problem in Hanoi, Vietnam. In addition to the considerations of the standard VRPTW, they consider a landfill operation that is the dumping of the collected garbage at the landfill, and inter-arrival time constraints between two consecutive visits at a stop. They

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incorporated the landfill operation by assuming that a vehicle starts a new route from the depot after landfill. Or-opt and 2-opt algorithms are adopted to improve the solution quality.

Poot *et al.* [6] presented several non-traditional constraints of real-life vehicle routing problems such as multiple capacity constraints, vehicle type constraints, region constraints, served first or last constraint, and multiple time windows. They proposed a savings-based method for the problems that have those constraints.

Angelelli and Speranza [7] addressed the periodic vehicle routing problem with intermediate facilities (PVRP–IF). When a vehicle visits an intermediate facility, its capacity will be renewed. They proposed a tabu search algorithm with four move operators: move a customer in the same day, change the visiting schedule, redistribution of customers, and simplification of intersection. Initial solutions are built by choosing a visiting schedule randomly to each customer and constructing vehicle routes on each day using iterative insertion procedure. Angelelli and Speranza [8] applied their algorithm for estimating the operating costs of different waste-collection systems: traditional system with three-man crew, side-loader system, and side-loader system with demountable body. The differentiator between their problem and ours is the time windows of the stops and the facilities. Our problem requires explicit consideration of time windows.

Teixeira et al. [9] applied a heuristic approach for a PVRP for the separate collection of three types of waste: glass, paper, and plastic/metal. The approach has three phases: define a zone for each vehicle, define waste type to collect on each day, and select the sites to visit and sequence them. Eisenstein and Iyer [10] used a Markov decision process to model the residential waste collection problem in the city of Chicago. They modeled the weight and time required to collect waste from a city block as normally distributed random variables. Action in their Markov decision process is the choice of route that visits the dumpsite once or twice. Chang et al. [11] discuss how combining GIS functions with analytical models can help analyzing alternative solid waste collection strategies for a metropolitan city in Taiwan. Mourao and Almeida [12] model the residential garbage collection problem in a quarter of Lisbon, Portugal as a capacitated arc routing problem, and propose two lower-bounding methods and a route-first, cluster-second heuristic method.

Kim *et al.* [13] address a real life waste collection VRPTW with consideration of multiple disposal trips and drivers' lunch breaks. They address the problem by using an extension of Solomon's well-known insertion approach. Ombuki-Berman *et al.* [14] address the same problem by using a multi-objective genetic algorithm on a set of benchmark data from real-world problems obtained by Kim *et al.* [13].

Benjamin and Beasley [15] improved the results when minimizing travel distance using a tabu search and variable neighborhood search and a combination of these. A very similar problem, with only one disposal site, is addressed by Tung & Pinnoi [5], where they modified Solomon's insertion algorithm and apply it to a waste collection problem in Hanoi, Vietnam. Nuortio *et al.* [16] present a guided variable neighborhood thresholding metaheuristic for the problem of optimizing the vehicle routes and schedules for collecting municipal solid waste in Eastern Finland. Solid waste collection is furthermore considered by Li *et al.* [17] for the City of Porto Alegre, Brazil. Their problem consists of designing daily truck schedules over a set of previously defined collection trips, on which the trucks collect solid waste in fixed routes and empty loads in one of several operational recycling facilities in the system. They used a heuristic approach to solve the problem. Buhrkal *et al.* [18] study the Waste Collection Vehicle Routing Problem with Time Window which is concerned with finding cost optimal routes for garbage trucks such that all garbage bins are emptied and the waste is driven to disposal sites while respecting customer time windows and ensuring that drivers are given the

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breaks that the law requires. they propose an adaptive large neighborhood search algorithm for solving the problem and illustrate the usefulness of the algorithm by showing that the algorithm can improve the objective of a set of instances from the literature as well as for instances provided by a Danish garbage collection company.

3 Problem formulation

In this section we formally define the WCVRP. The problem is defined on a graph where the set of nodes $N = \{1,\ldots,n\}$ consists of a depot and a disposal site which are consider as one node $\{1\} \in N$, $n-1$ customers $\{2,\ldots,n\} \in N$ and the set of arcs is $G = \{(i,j) \mid i,j \in N, i \neq j\}$. Let M be the set of vehicles and let K be the set of types of waste. It is assumed that all vehicles can have different capacity O_m. The objective of the WCVRP is to find a set of routes for the vehicles, minimizing total travel cost and satisfying vehicle capacity, such that all customers are visited exactly once and rout of each vehicles, is chosen so that the similar trashcan be placed in one rout. For this purpose, similar pattern was designed based on the probability of presence of the type of waste in trashcan P_{ki}, and dissimilarity is shown as a penalty in the objective function.

The problem can then be modelled using one type of variables: r_{ijm} is one if and only if vehicle $m \in M$ uses arc $(i,j) \in G$. A mathematical model for the WCVRP is:

$$Min\ C(x) = \sum_{(i,j)\in G}\left(C_{ij} + \sum_{k=1}^{K}\left|P_{ki} - P_{kj}\right|\right).\sum_{m=1}^{M} r_{ijm} + \left(\sum_{m=1}^{M} E_m + \sum_{m=1}^{M}\left(\sum_{j=2}^{N} r_{1jm}.LS_m\right)\right)$$

s.t.

$$\sum_{j=2}^{N} r_{1jm} = 1, \quad \forall m \in M, \tag{1}$$

$$\sum_{i=2}^{N} r_{i1m} = 1, \quad \forall m \in M, \tag{2}$$

$$\sum_{i=1}^{N}\sum_{m=1}^{M} r_{ijm} = 1, \quad \forall j \in N \setminus \{1\},\ i \neq j, \tag{3}$$

$$\sum_{i=2}^{N} r_{ijm} = \sum_{k=2}^{N} r_{jkm}, \quad \forall j \in N \setminus \{1\},\ m \in M, \tag{4}$$

$$\sum_{(i,j)\in G} r_{ijm}.ld_i \leq O_m, \quad \forall m \in M, \tag{5}$$

$$\sum_{(i,j)\in G} r_{ijm}.ld_i \geq 0, \quad \forall m \in M, \tag{6}$$

$$r_{iim} = 0, \quad \forall i \in N, m \in M, \tag{7}$$

$$r_{ijm} + r_{jim} \leq 1, \quad \forall i,j \in N \setminus \{1\},\ m \in M, \tag{8}$$

$$z_{ij} = \begin{cases} 1 & \sum_{k=1}^{K}\left|P_{ki} - P_{kj}\right| \leq \alpha \\ 0 & O.W \end{cases}, \tag{9}$$

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$$z_{1j} = 1, \quad \forall j \in N, \tag{10}$$

$$z_{i1} = 1, \quad \forall i \in N, \tag{11}$$

$$d_{ij} = \begin{cases} 1 & L_{ij} \le \beta \\ 0 & O.W \end{cases}, \tag{12}$$

$$r_{ijm} \le d_{ij} + z_{ij}, \quad \forall m \in M, \tag{13}$$

$$\sum_{k=1}^{K} P_{ki} = 1, \quad \forall i \in N, \tag{14}$$

$$r_{ijm} \in \{0,1\}, \quad \forall (i,j) \in G, m \in M. \tag{15}$$

The objective function minimizes the travel cost under the restriction of the following constraints. All m vehicles must leave (1) and return (2) to the depot. Constraint (3) ensures that all customers are serviced exactly once. Inflow and outflow must be equal except for the depot nodes (4). Vehicle capacity is given by (5) and (6). Constraints (7) and (8) are for the amount of binary variables. How to calculate the parameters of the similarity and proximity of trashcans (z_{ij} and d_{ij}) is shown in the Constraints (9), (10), (11) and (12). Constraint (13) shows the balance between similarity and proximity of trashcans for routing. Constraint (14) shows that P_{ki} is a possibility. Finally (15) imposes binary variables.

4 Computational results

In this section, we solved the model by using LINGO 0.9 software and analyze the output results of solved model. Table 1 shows the results of solved model for big size in which the value of index is: M=7, K=7 and n=9. Objective function value is equal to 55964.60 which have obtained after 3 minutes.

Table 1 Results of solved model by LINGO 9.0

Variable	Value
R(1, 2, 4)	1.000000
R(1, 3, 1)	1.000000
R(1, 4, 2)	1.000000
R(1, 5, 5)	1.000000
R(1, 6, 7)	1.000000
R(1, 8, 6)	1.000000
R(1, 9, 3)	1.000000
R(2, 1, 4)	1.000000
R(3, 1, 1)	1.000000
R(4, 1, 2)	1.000000
R(5, 1, 5)	1.000000
R(6, 7, 7)	1.000000
R(7, 1, 7)	1.000000
R(8, 1, 6)	1.000000
R(9, 1, 3)	1.000000

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In this example, we assume that there are 8 trashcans and one depot in an area. The presence of 7 types of waste in the trashcan is clear. Also there are 7 different vehicles for collecting wastes. These vehicles should service the trashcans so that where possible, similar trashcans have serviced by one vehicle to the quality of wastes not diminished because of waste combination. According to output results in Table 1, let us consider the nonzero decision variable corresponding to the seventh vehicle: R(1, 6, 7), R(6, 7, 7) and R(7, 1, 7). These variables indicates that the 7th vehicle began to move from node 1 (depot) to node 6 in order to service it. And then move to node 7 and finally, it returns to node 1 (depot) after giving service to node 7. We can see the route of 7th vehicle in these VRP network in Fig. 2.

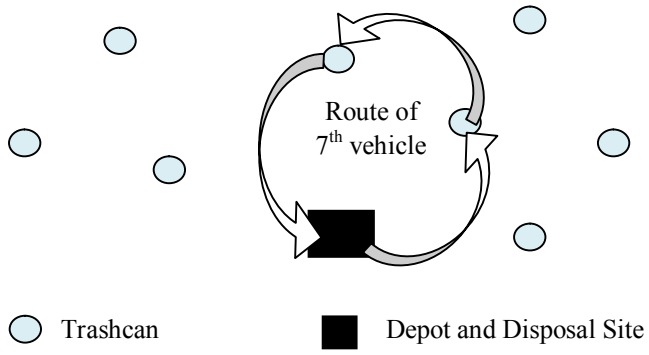

Fig. 2 Route of 7th vehicle in VRP network

5 Conclusions

In this paper we considered the Waste Collection VRP. A mathematical modeling formulation is given for the general WCVRP with consideration of similarity of the trashcans in terms of the types of waste which is in the trashcans in order to maintain the quality of wastes. The vehicles should service the trashcans so that where possible, similar trashcans have serviced by one vehicle to the quality of wastes not diminished because of waste combination. Finally the WCVRP has been solved using the LINGO software and the output results, has been analyzed.

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