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# Two Stage Flow Shop Scheduling Problem Including Transportation Time and Weightage of Jobs with Branch and Bound Method

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**Abstract** This paper presents an algorithm with the help of branch and bound approach for the two stage flow shop scheduling problem including transportation time and weightage of jobs; processing times are associated with their respective probabilities. Our main objective is to find the optimal/ near optimal sequence of jobs in order to minimize the total elapsed time. An algorithm is clarified with the help of a numerical example.

**Keywords** Flow Shop Scheduling, Branch and Bound Method, Elapsed Time, Transportation Time, Weightage of Jobs.

## 1 Introduction

In the context of manufacturing, scheduling is fundamentally related to the problem of finding a successive assignment of limited resources to a number of jobs which is optimal in terms of certain performance measures. On many occasions in manufacturing environments, a set of processes is needed to be serially performed in several stages before a job is completed. Such systems are referred to as flow shop environments. In a flow shop system, a set of n different jobs are needed to be processed on a sequential set of m machines. That is, each job consists of m operations where each operation must be performed on a different machine for an amount of processing time. Each machine can handle only one job at a time, and the operation of a machine on a job usually cannot be pre-empted. The research into flow shop problems has drawn a great attention in the last decades with the aim to decrease the cost and to increase the effectiveness of industrial production. Johnson [1] gave a procedure to obtain the optimal sequence for n-jobs, two – three machines flow shop scheduling problem with an objective to minimize the makespan. The work was developed by Ignall and Scharge [2], Brown and Lomnicki [3], Bagga [4], Smith, et at [5], Gupta, [6], Maggu and Das [7], Yoshida and Hitomi [8], Singh, [9], Chander Sekharan [10], Anup [11], Gupta Deepak [12], Lomnicki, [13], Chandermouli [14] etc. by considering the various parameters.

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The weight of job shows the relative priority over some other jobs in a schedule of jobs. Higher the weight a job has, the more important it becomes for processing in comparison with other jobs in the operating schedule. The scheduling problems with weights arise when inventory costs for jobs are involved. Further the scheduling problem which does not involve "weight" of job is called "simple or the unweighted scheduling problem", whereas the scheduling problem involving "weight" of jobs is referred to as "weighted scheduling problem".

There are practical scheduling situations when certain times are required by jobs for transportation from one machine to another machine. This situation can be visualized when the machines on which jobs are to be processed are planted at different places, and these jobs require additional times in the forms of loading-time of jobs, moving time of jobs and then unloading time of jobs. The sum of all these times is known as transportation time of jobs. This paper studies the two stage flow shop scheduling problem introducing the weightage of jobs and transportation time. This makes the problem wider and more practical in process/ production industry.

**Assumptions:**

1. No passing is allowed.
2. Each operation once started must be performed till completion.
3. Jobs are independent of each other.
4. A job is entity, i.e. no job may be processed by more than one machine at a time.

**Notations:**

We are given n jobs to be processed on a three stage flowshop scheduling problem, and we have used the following notations:

| | | |
|---|---|---|
| $a_i$ | : | Processing time for $i^{th}$ job on machine A |
| $b_i$ | : | Processing time for $i^{th}$ job on machine B |
| $p_i$ | : | Probability associated to the processing time $a_i$. |
| $q_i$ | : | Probability associated to the processing time $b_i$. |
| $A_i$ | : | Expected Processing time for $i^{th}$ job on machine A |
| $B_i$ | : | Expected Processing time for $i^{th}$ job on machine B. |
| $t_i$ | : | Transportation time from machine A to machine B. |
| $C_{ij}$ | : | Completion time for job $i^{th}$ on machines A and B |
| $S_0$ | : | Optimal sequence |
| $J_r$ | : | Partial schedule of r scheduled jobs |
| $J_{r'}$ | : | The set of remaining (n-r) free jobs |

## 2 Mathematical Development

Consider n jobs say i=1, 2, 3 … n are to be processed on two machines A & B in the order AB. Let $a_i$ and $b_i$ be the processing times of $i^{th}$ job on machine A and machine B with probability $p_i$ and $q_i$ respectively. $A_i$ and $B_i$ be the expected processing times of $i^{th}$ job on each machine A & B respectively. Let $t_i$ be the transportation time of $i^{th}$ job from machine A to machine B, and Let $w_i$ be the weightage of $i^{th}$ job.

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The mathematical model of the problem in matrix form can be stated as:

**Table 1** The mathematical model of the problem in matrix form

| job | Machine A | | $t_i$ | Machine B | | weight |
|---|---|---|---|---|---|---|
| i | $a_i$ | $p_i$ | | $b_i$ | $q_i$ | $w_i$ |
| | $a_1$ | $p_1$ | $t_1$ | $b_1$ | $q_1$ | $w_1$ |
| | $a_2$ | $p_2$ | $t_2$ | $b_2$ | $q_2$ | $w_2$ |
| 1 | | | | | | |
| 2 | $a_3$ | $p_3$ | $t_3$ | $b_3$ | $q_3$ | $w_3$ |
| 3 | | | | | | |
| - | - | - | - | - | - | - |
| - | | | | | | |
| n | - | - | - | - | - | - |
| | $a_n$ | $p_n$ | $t_n$ | $b_n$ | $q_n$ | $w_n$ |

Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time, using branch and bound technique.

## 3 Algorithm

Step 1: Calculate
(i) $A_i = a_i \times p_i$ (ii) $B_i = b_i \times q_i$

Step 2: (i) $G_i = A_i + t_i$ and (ii) $H_i = B_i + t_i$

Step 3: Compute Minimum ($G_i$, $H_i$)

1) If Min ($G_i$, $H_i$) = $G_i$ then define $G_i' = G_i + w_i$ and $H_i' = H_i$
2) If Min ($G_i$, $H_i$) = $H_i$ then define $G_i' = G_i$ and $H_i' = H_i + w_i$

Step 4: Define a new reduced problem with $G_i''$ and $H_i''$ where

$G_i'' = G_i' / w_i$ and $H_i'' = H_i' / w_i$

Step 5: Calculate

(i) $l_1 = t(J_r, 1) + \sum_{i \in J_r'} G_i'' + \min_{i \in J_r'}(H_i'')$

(ii) $l_2 = t(J_r, 2) + \sum_{i \in j_r'} H_i''$

Step 6: Calculate $l = \max(l_1, l_2)$

Evaluate $l$ for the n classes of permutations, i.e., for these starting with 1,2,…n respectively. Explore the lowest lower bound vertex for (n-1) subclasses and again concentrate on the lowest label vertex. Thus, we get the optimal/near optimal sequence.

Step 7: Prepare in-out table for the optimal sequence obtained in step 6 and get the minimum

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total elapsed time.

## 4 The Numerical Illustration

Consider 5 jobs 2 machine flow shop problem whose processing time of the jobs on each machine, Transportation time $t_i$ and weights of jobs $w_i$ are given in Table 2.

**Table 2** Processing time, transportation time, and weights of jobs of the problem

| job | Machine A | | $t_i$ | Machine B | | weight |
|---|---|---|---|---|---|---|
| i | $a_i$ | $p_i$ | | $b_i$ | $q_i$ | $w_i$ |
| 1 | 50 | 0.3 | 4 | 105 | 0.2 | 2 |
| 2 | 180 | 0.1 | 3 | 170 | 0.1 | 1 |
| 3 | 80 | 0.2 | 5 | 75 | 0.2 | 3 |
| 4 | 70 | 0.3 | 2 | 70 | 0.2 | 4 |
| 5 | 190 | 0.1 | 6 | 60 | 0.3 | 2 |

Our objective is to obtain an optimal schedule for above said problem in order to minimize the total elapsed time.

**Solution:** As per Step 1: Expected processing times are as in Table 3:

**Table 3** Expected processing times of step 1

| job | Machine A | $t_i$ | Machine B | weight |
|---|---|---|---|---|
| i | $A_i$ | | $B_i$ | $w_i$ |
| 1 | 15 | 4 | 21 | 2 |
| 2 | 18 | 3 | 17 | 1 |
| 3 | 16 | 5 | 15 | 3 |
| 4 | 21 | 2 | 14 | 4 |
| 5 | 19 | 6 | 18 | 2 |

As per Step 2: Reduced problem is in Table 4:

**Table 4** The reduced form of the problem

| job | Machine A | Machine B | weight |
|---|---|---|---|
| i | $G_i$ | $H_i$ | $w_i$ |
| 1 | 19 | 25 | 2 |
| 2 | 21 | 20 | 1 |
| 3 | 21 | 20 | 3 |
| 4 | 23 | 16 | 4 |
| 5 | 25 | 24 | 2 |

As per Step 3 &4: Expected Weighted times are as in Table 5:

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**Table 5** Expected weighted times of steps 3 and 4

| job | Machine A | Machine B |
|---|---|---|
| i | $G_i''$ | $H_i''$ |
| 1 | 10.5 | 12.5 |
| 2 | 21 | 21 |
| 3 | 7 | 7.6 |
| 4 | 5.75 | 5 |
| 5 | 12.5 | 13 |

As per Step 5 & 6: The lower bounds are as in Table 6:

**Table 6** The lower bounds of steps 5 and 6

| Node | LB(Jr) |
|---|---|
| 1 | 69.5 |
| 2 | 80.09 |
| 3 | 66.09 |
| 4 | 64.84 |
| 5 | 71.5 |
| 41 | 70.34 |
| 42 | 80.8 |
| 43 | 69.25 |
| 45 | 72.34 |
| 431 | 69.75 |
| 432 | 80.25 |
| 435 | 71.75 |
| 4312 | 78.25 |
| 4315 | 77.75 |

Thus, the optimal sequence is 4-3-1-5-2

As per Step 7: The flow time table for the optimal sequence is as follows:

**Table 7** The flow time table for the optimal sequence

| job | Machine A | $t_i$ | Machine B | weight |
|---|---|---|---|---|
| i | In – Out | | In - Out | $w_i$ |
| 4 | 0 -21 | 2 | 23 – 37 | 4 |
| 3 | 21 – 37 | 5 | 42 – 57 | 3 |
| 1 | 37 – 52 | 4 | 57 – 78 | 2 |
| 5 | 52 – 71 | 6 | 78 – 96 | 2 |
| 2 | 71 – 89 | 3 | 96 – 113 | 1 |

Hence, for the optimal sequence 4-3-1-5-2, the total elapsed time is 113 units.

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