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An Evaluation of an Adaptive Generalized Likelihood Ratio Charts for Monitoring the Process Mean

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Abstract When the objective is quick detection both small and large shifts in the process mean with normal distribution, the generalized likelihood ratio (GLR) control charts have better performance as compared to other control charts. Only the fixed parameters are used in Reynolds and Lou's presented charts. According to the studies, using variable parameters, detect process shifts faster than fixed parameter control chart. In this paper, the performance of the adaptive GLR chart is evaluated. Based on the study, it is shown that the variable sampling size and sampling interval (VSSI) is more effective than the other adaptive GLR control charts in detecting small process mean shifts.

Keywords: Average Time to Signal, Adaptive Control Chart, Generalized Likelihood Ratio, Variable Parameters.

1 Introduction

Statistical process control is a powerful tool in creating stability and improving process effectiveness, via reducing variability. Control charts are the strongest tools in this regard, used for monitoring the processes in definite time and for detecting the special cause of variation. The most important charts for monitoring the mean processes, where the process observations are assumed to be independent normal random variables, include Shewhart control charts, cumulative sum (CUSUM) charts and exponentially weighted moving average (EWMA). The traditional Shewhart control chart $\bar{x}$ is effective if the size of the shift in μ is large, but is not effective if the size of shift is small. CUSUM and EWMA charts can be tuned to be very effective for detecting small shifts in μ, but then these charts will not be very effective for detecting large shifts. Since in application, the size of shifts in μ that occurs will be unknown, the charts are needed to be more effective in determining a wide range of shifts.

One option to obtain better performance in detecting a wide range of shift sizes is combining two or more control charts. The combinations of Shewhart control chart and CUSUM by Lucas [1] and two or more CUSUM charts by Stoumbos and Reynolds [2] could

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be indicated in this regard. In addition to increasing the number of charts, the other disadvantage of the control charts combination is their complexity in determining the parameters and designing the charts for the users.

The other method is adaptive control charts. At least one parameter in these charts (sampling size "n", sampling interval "d", control limit "k") is variable. By drawing the warning limits (w) on the chart, this chart is divided into different regions and due to the current value of the control statistic, the relevant control chart parameters are defined for the next sampling activity. When the current statistic is placed in the warning region, larger sample size, smaller interval, and narrower control limits are used, in adaptive control charts with variable parameters and when the data is located in the central area, smaller sample size, larger time interval and wider limits are used.

Much work on developing adaptive control charts has been performed for monitoring the mean and /or variance from independent normal processes. Variable sampling interval (VSI) Shewhart charts include Reynolds and Arnold [3] and Reynolds *et al*. [4]. Reynolds *et al*. [3] investigated VSI CUSUM charts, and Shamma, *et al*. [5], Saccucci, *et al*. [6]. Variable sampling size (VSS) Shewhart charts was indicated by Park and Choi [7], Prabhu, *et al*. [8], Zimmer *et al*. [9]. Annadi, *et al*. [10] investigated VSS CUSUM charts. Costa [11] indicated another adaptable model in which all the variable parameters (VP) were considered. All the three control chart parameters, i.e. sample size, sampling interval and control limits are variables in the latter chart. He showed that this chart has a better performance as compared to Shewhart control charts with constant parameters, VSS $\bar{X}$, VSI $\bar{X}$ and VSSI $\bar{X}$ parameters, in detecting small to medium shifts.

The other option is according to likelihood ratio test, which is usually referred to as generalized likelihood ratio (GLR). Reynolds and Lou [12] showed that this chart has very good performance in determining a wide range of mean changes. Designing these charts is also practically easy, since users do not need to determine the control chart parameters, apart from the control limits, and the limits are stated in a table by Reynolds and Lou, with regards to the false alarm rate and the window size. Fixed control limits, sampling size, and intervals are used in the presented chart by Reynolds and Lou. Peng *et al* [13] showed that generalized likelihood ratio control charts with variable intervals (VSI GLR) have better performances.

According to the advances in sampling techniques, control charts with variable parameter are considered a lot and the studies in this regard show that the performance of the charts is much better than control charts with fixed parameter for the quick detection of shifts. No such studies are done for the generalized likelihood ratio chart, so using variable control limits, variable sampling size and variable sampling interval (one or more than one variable parameter) could considerably improve the performance of the generalized likelihood ratio chart.

Adaptive (VSS, VSSI, and VP) Generalized likelihood ratio control charts are considered in this paper and this chart is compared with GLR control charts and VSI GLR charts. Adaptive GLR control chart, performances measuring indices and adaptive GLR control chart design are considered and discussed in the 2nd to 5th sections, respectively. The performance of adaptive GLR charts (VSI, VSS, VSSI and VP) are compared together for the changes in the mean in the section 5, and a numerical example is given in section 6. Finally, the conclusion and proposals are observed in the last part of the paper.

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2 Adaptive GLR Control Charts with Variable Parameters

Suppose that the variable x being monitored in the process has a normal distribution with mean μ and variance σ^2. The in-control values μ_0 and σ_0^2 are assumed to be known or have been accurately estimated during a Phase I period. The objective is to quickly detect a wide range of two-sided shifts in μ_0 to μ_1.

Assume that the samples are taken independently at each sample point. The sampling intervals, sample size, and control limits are the functions of the current value of chart statistic, in VP GLR charts. Only two intervals (d_1,d_2), two sample sizes (n_1,n_2) and two GLR control limits (h_{1GLR},h_{2GLR}) are used in this study, that $d_2 \prec d_0 \prec d_1$, $n_1 \prec n_0 \prec n_2$ and $h_{2GLR} \prec h_{0GLR} \prec h_{1GLR}$ and n_0,d_0 *and* h_{0GLR} are the parameters of standard GLR or GLR with fixed parameters. When we $h_{1GLR}=h_{0GLR}=h_{2GLR}$, $n_1=n_0=n_2$ and $d_1=d_0=d_2$ we have the standard GLR chart with fixed parameters (FP GLR chart). when $h_{1GLR}=h_{0GLR}=h_{2GLR}$, $n_1 \prec n_0 \prec n_2$ and $d_2 \prec d_0 \prec d_1$ the VP GLR chart is called GLR chart with variable sampling size and sampling intervals (VSSI GLR chart). When $h_{1GLR}=h_{0GLR}=h_{2GLR}, n_1=n_0=n_2$ and $d_2 \prec d_0 \prec d_1$ the VP GLR chart is called GLR chart with variable sampling intervals (VSI GLR chart). When $h_{1GLR}=h_{0GLR}=h_{2GLR}$, $n_1 \prec n_0 \prec n_2$ and $d_1=d_0=d_2$ the VP GLR chart is called GLR chart with variable sampling size (VSS GLR chart).

To avoid using two with GLR charts for small and large sizes, a GLR chart with two scaled vertical axes could be used (Fig. 1), where the left axis is for the GLR chart d_1, n_1, h_{1GLR} parameters and the warning limit (w_1), and the right axis is for the GLR chart d_2,n_2,h_{2GLR} parameters and the warning limit (w_2) and the horizontal axis (time) is unique for each chart.

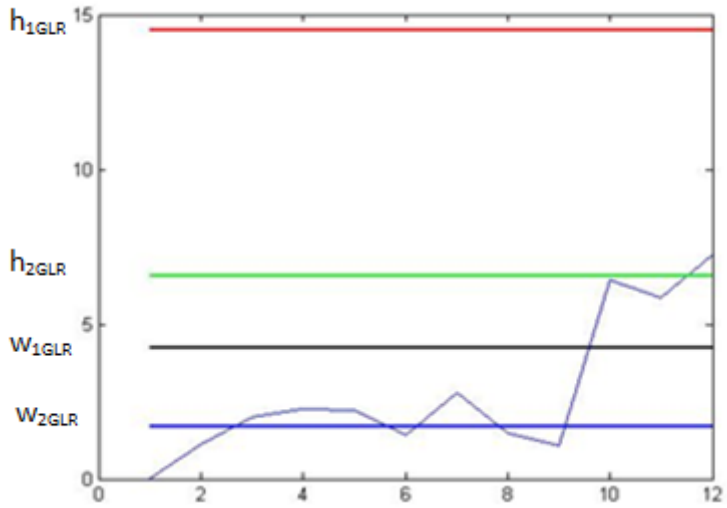

Fig. 1 Adaptable GLR chart (V_p GLR)

Suppose that at each sampling point, samples of $n \geq 1$ are taken from the process. Let $X_i=(X_{i1},X_{i2},....,X_{in})$ represent the data vector at sampling point i, and let X_i denote the sample mean at sampling point i, for i=1, 2,... It should be pointed out that when n = 1, this vector X_i reduces to the scalar X_i, and the ith sample mean X_i is equal to $\bar{X}_i$.

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At sampling point k, we have the data $X_1, X_2, ..., X_K$. Consider the null hypothesis that there has been no shift in the mean versus the alternative hypothesis that a mean shift from μ_0 to μ_1 has occurred at some time between samples τ and $\tau + 1$, where $\mu_1 \neq \mu_0$ and $\tau \prec k$. The likelihood functions under the null hypothesis and alternative hypothesis, respectively. A log likelihood ratio test statistic can be written as:

$$R_k = ln \frac{max 0 \le \tau < k \; L(\tau, \mu_1 | \bar{x}_1, \bar{x}_2, .., \bar{x}_k)}{L(\infty, \mu_0 | \bar{x}_1, \bar{x}_2, .., \bar{x}_k)} = ln \frac{max \; 0 \le \tau < k \prod_{s=1}^{\tau} \frac{1}{\sqrt{2\pi}\,\sigma_0} e^{\frac{-n_i(\bar{x}_i - \mu_0)^2}{2\sigma_0^2}} \prod_{i=\tau+1}^{k} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{n_i(\bar{x}_i - \mu_1)^2}{2\sigma_0^2}}}{\prod_{i=1}^{k} \frac{1}{\sqrt{2\pi}\sigma_0} e^{\frac{-n_i(\bar{x}_i - \mu_0)^2}{2\sigma_0^2}}} \quad (1)$$

Since, In this chart, different sample size are used, therefore $\bar{x}_i$ have not equal distribution and there distributions depends on sample size, to remove this problem first standardizing each sample mean using the corresponding sample size. This gives the standardized sample mean z_k given by (2) and then the likelihood ratio will be written in which, instead of $\bar{x}$ variable, z variable is used, so that, R_k will be written by (3):

$$z_i = \sqrt{n_i} \frac{\bar{x}_i - \mu_0}{\sigma_0} \quad (2)$$

$$R_k = ln \frac{max 0 \le \tau < k \; L(\tau, M_1 | z_1, z_2, .., z_k)}{L(\infty, M_0 | z_1, z_2, .., z_k)} = ln \frac{max 0 \le \tau < k \prod_{s=1}^{\tau} \frac{1}{\sqrt{2\pi}} e^{\frac{-z_i^2}{2}} \prod_{i=\tau+1}^{k} \frac{1}{\sqrt{2\pi}} e^{-\frac{(z_i - M_1)^2}{2}}}{\prod_{i=1}^{k} \frac{1}{\sqrt{2\pi}} e^{\frac{-z_i^2}{2}}} \quad (3)$$

$$R_k = max_{0 \le \tau < k} \frac{(k-\tau)}{2} (\hat{M}_{1,\tau,k})^2$$

Where $\hat{M}_{1,\tau,k}$ is the maximum likelihood estimator of M_1,

$$\hat{M}_{1,\tau,k} = \frac{\sum_{i=\tau+1}^{k} z_i}{k-\tau} \quad (4)$$

Regarding equation (3), it is required to compute R_k for all the given past data. But, in case of changes in the time intervals, the real time of the changes in the observations is near (k) in "m" observations before that, the maximum value is only occurred in "m" previous samples. The best performance in determining the real changing time is occurred when the value of (m) is large enough. Reynolds *et al*. have suggested m=400. By Considering the "m" previous observations, equation (3) could be written as equation (5):

$$R_k = max_{\; max(0,k-m) \le \tau < k} \frac{(k-\tau)}{2} (\hat{M}_{1,\tau,k})^2 \quad (5)$$

For the decision rule for VP GLR, it is required to determine the values for (w_1, w_2) and (h_{1GLR}, h_{2GLR}); the values for w_1 and w_2 are warning limits and the values for h_{1GLR}, h_{2GLR} are control limits. After determining the values for these ranges, the first sample is taken

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randomly and the control statistic is calculated. If the value is located in the warning region of the left chart, the next sample will be taken with sample size (n_2) and the time interval (d_2). Then, it is compared with the warning region and control limit at the right chart. If it is not located in the warning region, the next sample with the size (n_1) and interval (d_1) will be taken and it will be compared to the warning limits of the left side chart. As a whole, when the control statistic is placed in the warning region of any of the charts, the next sample will be taken with the sample size of (n_2) and time interval (d_2), to be compared with the limits (w_2) and (h_{2GLR}), otherwise the next sample with sample size (n_1) and time interval (d_1) will be compared with the limits (w_1) and (h_{1GLR}). If in using the charts, the control statistic exceeds from the existing control limits, the process will be considered to be out of control.

3 Performance Metrics

The statistical performance of an adaptive GLR control chart can be evaluated by considering the number of samples to signal, the number of individual observations to signal, and the time required to signal. Let the Average Number of Samples to Signal (ANSS) be the expected number of samples from the start of monitoring at time $t_0 = 0$ to the time that the chart signals. Similarly, define the Average Number of Observations to Signal (ANOS) to be the expected number of individual observations from t_0 to the time that the chart signals. Also, define the Average Time to Signal (ATS) to be the expected length of time from t_0 to the time that the chart signals. The ANSS computed for $\mu = \mu_0$ is a measure of the average false alarm rate per sample, and the ATS computed for $\mu = \mu_0$ is a measure of the average false alarm rate per unit time. The ATS computed for $\mu = \mu_1$ is an appropriate measure of the chart's ability to detect a shift to μ_1 if the process starts out with $\mu = \mu_1$ at time $t_0 = 0$. However, in many applications, the process may start with $\mu = \mu_0$ and then shift to μ_1 at some random time in the future. The expected time required after the shift for the control chart to signal is called the Steady State ATS (SSATS). Similarly, if it is desirable to find the expected number of samples or the expected number of observations from the shift in μ to the signal, then a Steady State ANSS (SSANSS) or a Steady State ANOS (SSANOS), respectively, can be computed.

The SSATS values are used in this paper for measuring the performance of adaptive GLR chart and all these charts have similar ATS. According to Peng *et al*., The control limits are obtained for all the charts, by using stimulated data with standard normal distribution and the considered ATS values. The state of out of control is occurred between the samples “400” and “401” and the SSATS values for the simulated data are computed. The scale of changes is defined as $\delta = \frac{|\mu_1 - \mu_0|}{\sigma_0}$, the values are considered between 0.25 and 3.0 and m=400. All evaluation of the ATS and SSATS values were performed using 10000 iterations. The window size m= 400 is used as recommended by Reynolds and Lou.

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4 Design of Adaptive GLR Charts with Variable Parameters

When working with VP GLR control chart, a proportions of samples which have been taken with n_1 and n_2 sizes, should be cleared, define ψ_{n_1} and ψ_{n_2}, respectively, to be expected sample size taken with the size of n_1 and n_2 before the signal when process is in control, the ANSS , ANOS and ATS can be written in terms ψ_{n_1} and ψ_{n_2} as:

$$ANNS = 1 + \psi_{n_1} + \psi_{n_2} \tag{6}$$

$$ANOS = n_0 + n_1\psi_{n_1} + n_2\psi_{n_2} \tag{7}$$

$$ATS = d_0 + d_1\psi_{n_1} + d_2\psi_{n_2} \tag{8}$$

Let

$$p_{n1} = \frac{\psi_{n1}}{ANSS} \tag{9}$$

be the proportion of samples before the signal that specify that n_1 be used. From (6), (7) and (8), it follows that the average sample size $\bar{n}$ can be expressed as:

$$\bar{n} = \frac{ANOS}{ANSS} = \frac{n_0 + n_1\psi_{n_1} + n_2\psi_{n_2}}{ANSS} = n_1 p_{n_1} + \frac{n_2(ANSS - 1 - \psi_{n_1})}{ANSS} + \frac{n_0}{ANSS}$$

$$= n_1 p_{n_1} + n_2(1 - p_{n_1}) + \frac{n_0 - n_2}{ANSS} \tag{10}$$

And from (6), (8) and (9), it follows that the average sampling interval $\bar{d}$ can be expressed as:

$$\bar{d} = \frac{ATS}{ANSS} = d_1 p_{n_1} + \frac{d_2(ANSS - 1 - \psi_{n_1})}{ANSS} + \frac{d_0}{ANSS}$$

$$d_1 p_{n_1} + d_2(1 - p_{n_1}) + \frac{d_0 - d_2}{ANSS} \tag{11}$$

Since process start with $\mu = \mu_0$ and then shift to μ_1 at some random time in the future, the value of $\frac{n_0 - n_2}{ANSS}$ and $\frac{d_0 - d_2}{ANSS}$ can be neglected. In the design of chart, one must determine eight parameters (n_1, n_2), (d_1, d_2), (h_{1GLR}, h_{2GLR}) and (w_1, w_2), according to (10), (11) and costa (1999), these parameters must satisfy Equations (12) - (15) as follows:

$$n_0 = n_1 p_{n_1} + n_2(1 - p_{n_1}) \tag{12}$$

$$d_0 = d_1 p_{n_1} + d_2(1 - p_{n_1}) \tag{13}$$

$$p_{n_1} = p(R_k \prec w_i | R_k \prec h_{iGLR}, \sigma = \sigma_0, n = n_i) \tag{14}$$

$$p(R_k > h_{1GLR}) p_{n_1} + p(R_k > h_{2GLR})(1 - p_{n_1}) = p(R_k > h_{0GLR}) \tag{15}$$

where n_0 , d_0 and h_{0GLR} are the parameter of FP GLR chart. Usually n_0 = 3, 4 or 5, without losing generality one can set d_0 =1.

The three constraints in Equations (12), (13) allow the user to choose one of the pairs of parameters (n_1, n_2), (d_1, d_2) and then one parameter from each remaining pair. We recommended choosing the pair (n_1, n_2) and the elements d_2 and h_{1GLR} for two reasons:

(a) The range of feasible values for n_2 and d_2 depends on the time required to sample each item;

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(b)A VP chart is recommended to detect small shifts in the process mean. Under these conditions, the GLR charts work better when the false alarm risk α_1 is practically zero (that is, when h_{1GLR} is large).

In this paper, after determination of the h_{1GLR} ,in order to set the identical in – control ATS for the VP GLR charts the value of w_1 and h_{2GLR} obtained by simulation data. Since h_{GLR} is independent of the sample size, thus the value of w_1 and w_2 are independent of the sample size. Hence, the values of w_2 could be obtained by Equation (12) and according to Peng *et all* (2013).

5 Numerical Studies

Reynolds and Lou have shown that in the case of FP charts, the GLR chart has better overall performance across a wide range of shifts than standard Shewhart, CUSUM, or EWMA charts. Adaptive CUSUM and EWMA charts or combinations of two or more charts can have comparable overall performance to the GLR chart, but they are much more complicated. In this section, we evaluate the performance of the adaptive GLR chart and compare together. The main purpose of this section is to show the performance improvement that can be obtained going from the Fp GLR chart to the Vp GLR chart and compare the adaptive GLR charts together.

The average sampling interval is $\bar{d}=1$ and average sampling size is $\bar{n}=3$. We set that all charts in this section have the same in-control ATS of 740.8. The values SSATS with respect to an amount δ equal to 0.25, 0.5, 0.75, 1, 2, 3 with 10000 iterations. Table 1 gives the SSATS values for the FP GLR chart (in column [1]), VSI GLR (in column [2]), VSS GLR chart (in column [3]), VSSI GLR chart (in column [4]), and VP GLR charts (in columns [5]). It is also easy to see that overall, the adaptive GLR chart has much better performance than the FP GLR chart except for very large shifts Table 1 show that the VSSI GLR chart have smaller out -of- control SSATS value than other adaptive GLR charts when the process mean has the small shifts, which indicates that the chart has better ability to detect the small shifts. Thus, under consideration of false alarm rate and the detection ability, the VSSI chart a better choice for adaptive GLR charts.

Table 1 Steady-state average time to signal for the fixed parameters and adaptive generalized likelihood ratio chart

	FP GLR	VSI GLR	VSS GLR	VSSI GLR	VP GLR
n_1	3	3	2	2	2
n_2	3	3	9	9	9
d_1	1	1.15	1	1.15	1.15
d_2	1	0.1	1	0.1	0.1
h_{1GLR}	6.5548	6.5548	6.5548	6.5548	13.9894
h_{2GLR}	6.5548	6.5548	6.5548	6.5548	5.8593
w_1	-	3.0046	3.0046	3.0046	3.7835
w_2	-	3.004	3.0046	3.0046	2.8440
δ					
0.00	740.8	740.8	740.8	740.8	740.8

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	FP GLR	VSI GLR	VSS GLR	VSSI GLR	VP GLR
0.25	47.94	28.20	31.84	24.28	30.72
0.50	15.87	8.44	10.69	6.72	10.54
0.75	7.30	4.49	6.20	3.37	5.85
1.00	4.92	2.93	3.82	2.89	3.62
2.00	1.45	1.2	1.75	1.32	1.38
3.00	1.07	0.97	1.14	0.95	1.12

6 An example of designing the variable parameter generalized likelihood ratio chart

Consider an application in which an FP Shewhart $\bar{x}$ chart is being used based on taking samples of n = 4 every 2 hours. In this process we can also use, bigger sample size and shorter time intervals. But because of some technical limitations, minimum time is 15 minute ($d_2 = 0.25h$) and maximum sample size that can be derived from process is 6 ($n_2 = 6$).To significantly reduce the time to signal when the process is out-of-control, a VP GLR chart can be applied. Since three sigma control limits is used, so equal GLR chart has ATS=1481.6 and $d_0 = 2$, according to the Reynold and Lou table, value of h_{0GLR} is equal to 6.5385, by determining the values, $n_0 = 4,\ n_1 = 2,$ and $n_2 = 6,$ using (12), value of p_{n1}=0.50 and according to equation (13) value of d_1=3.75 can be obtained. And according to equation (15) and ATS=1481.6, $h_{1GLR} = 16.2025, h_{0GLR} = 6.5385$ value of $h_{2GLR} = 5.7706$. By using (14) and Peng, *et al*'s regression relation $w_1 = 3.7994$, $w_2 = 1.5463$ *and* $w_0 = 1.6591$ are obtained. Figures (2) and (3) respectively show Vp GLR and VSSI GLR charts with simulated data, the process was in control until time 20, then the shift of size $\delta = 0.75$ in μ was introduced.

According to figure (2), while process is out of control, for VP GLR chart, samples 21 and 22 with sample size 2 and time interval 3.75 hours have been taken, and sample 23 with sample size 6 and time interval 0.25 hours has been taken. A signal was generated after 7.75 hours after the change was occurred. According to figure (3), while process is out of control, for VSSI GLR chart, samples 21 with sample size 2 and time interval 3.75 hours have been taken, and sample 22, 23 and 24 with sample size 6 and time interval 0.25 hours has been taken. A signal was generated after 4.5 hours after the change was occurred

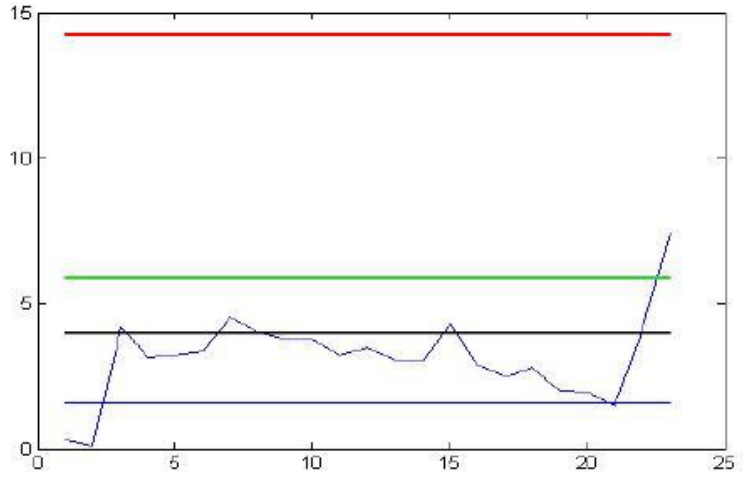

Fig. 2 Trace plot of the Variable parameter Generalized Likelihood Ratio statistic.

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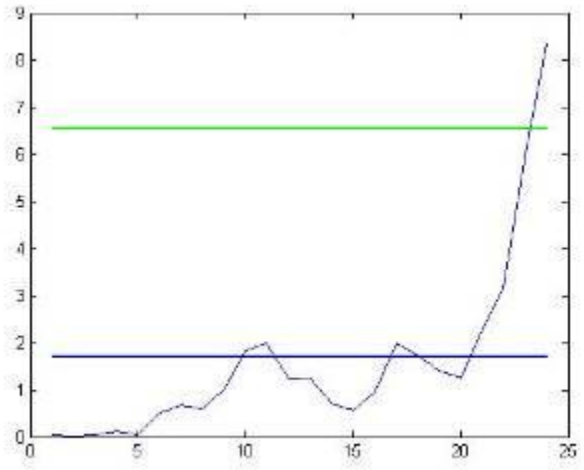

Fig. 3 Trace plot of the variable sampling size and sampling interval Generalized Likelihood Ratio statistic.

7 Conclusions

In this paper, we showed how to apply variable parameter scheme to GLR chart for monitoring the process mean. The fixed parameter GLR chart gives a better overall performance across a wide range of shifts than any single standard shewhart, CUSUM, or EWMA chart. The performance of the adaptive GLR chart has been investigated, and the results showed that VSSI GLR chart is better than other adaptive GLR charts. Therefore, we recommend that practitioners apply the VSSI GLR chart whenever the VSSI scheme is feasible in application. An important contribution of this paper is that it provides a design methodology for the VSSI GLR chart such that the VSSI GLR chart can be easily used. In future studies a similar approach could be applicable for monitoring the process variance and multivariable GLR control chart for monitoring the process mean.

References

1. Lucas, J. M., (1982). Combined Shewhart – CUSUM Quality Control Chart Schemes: A Robustness Study for CUSUM Quality Control Schemes. Journal of Quality Technology.14.pp.51-59.
2. Stoumbos, Z. G., Reynolds, M. R., Jr., (1996) Control charts applying a general sequential test at each sampling point. Sequential Analysis, 15, 159-183.
3. Reynolds, M. R., Jr., Arnold , JC, Runger, G. C., (2001). EWMA control chart with variable sample size and variable sampling interval IIE Transaction 33..s11s30
4. Reynolds, M. R., Jr., Amin, R. W., Arnold, J. C., Nachlas, J. A., (1988). X Charts with variable sampling intervals. Technometrics, 30, 181-192.
5. Shamma, S. E., Amin, R. W., Shamma, A. K., (1991). A double exponentially weighted moving average control procedure with variable sampling intervals. Communications in Statistics-Simulation and Computation, 20, 511-52.
6. Saccucci, M.S., Amin, R. W., Lucas, J. M., (1992). Exponentially weighted moving average control schemes with variable sampling intervals. Communications in Statistics-Simulation and Computation, 21, 627-657.
7. Park, C., Choi, K., (1993). Economic design of control charts with variable sample size scheme, in Proceedings, Section on Quality and Productivity, ASA.

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8. Prabhu, S. S., Montgomery, D. C., Runger, G. C., (1994). A combined adaptive sample size and sampling interval _X control scheme. Journal of Quality Technology, 26, 164-176.
9. Zimmer, L. S., Montgomery, D. C., Runger, G. C., (1998). Evaluation of a three-state adaptive sample size _X control chart. International Journal of Production Research, 36, 733-743.
10. Annadi, K., Runger, G. C., Montgomery, D. C., (1995). An adaptive sample size cusum control chart international journal of production economics, 33, 1605-166.
11. Costa, A. F. B., (1998). Joint _X and R charts with variable parameters. IIE Transactions, 30, 505-514.
12. Reynolds, M. R., Jr., Lou J., (2010). An evaluation of a GLR control chart for monitoring the process mean. Journal of Quality Technology, 42, 287–310.
13. Peng., Y., Xu.,l., Reynolds, M. R., Jr., (2013). The Design of the Variable Sampling Interval Generalized Likelihood Ratio Chart for Monitoring the Process Mean, Qual. Reliab. Eneng. Int.

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