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# Cross-Efficiency Evaluation by the Use of Ideal and Anti-Ideal Virtual DMUs' Assessment in DEA

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**Abstract** To rank Decision Making Units (DMU) in Data Envelopment Analysis (DEA), the peer-evaluation based cross-efficiency method is generally used. Indeed, in this method, each DMU is evaluated from the view point of other DMUs. In this article, a method is suggested which instead examines each DMU just by using the weights resulting from the evaluation of ideal and anti-ideal virtual DMUs and thus exhibits a new secondary goal that possibly prevents the existence of multiple weights in cross-efficiency evaluation, in addition to introducing a new cross-efficiency score. The advantage of this method over the others is that it needs less computations in determining the cross-efficiency score. To this end, some examples are illustrated which show how it differs with other methods.

**Keyword:** Data Envelopment Analysis, Cross-Efficiency, Ideal and Anti-Ideal Virtual DMUs, Secondary Goal, Ranking.

## 1 Introduction

Data Envelopment Analysis (DEA), as introduced in Charnes et al. [1], is a nonparametric linear programming based technique for measuring relative efficiencies of a homogeneous set of Decision Making Units (DMUs) with multiple inputs and outputs. For each DMU, it provides efficiency scores in the form of a ratio of a weighted sum of the outputs to a weighted sum of the inputs. One of the most appealing aspects of this methodology is that we do not need to a priori know exactly the values of the involved weights; DEA tries to estimate the units under evaluation in the best way possible. Most of the DEA models assign an efficiency score less than one to inefficient DMUs (in input orientation model). A weakness of DEA is that a considerable number of DMUs typically is characterized as efficient. Thus, DEA does not allow for a ranking of the efficient DMUs themselves. For more information regarding the new ranking methods, the interested readers can refer to Hosseinzadeh Lotfi et al. [2], Jahanshahloo and Shahmirzadi [3], Nasseri et al. [4] and Emrouznejada et al. [5].

Cross-efficiency evaluation is an approach for ranking DMUs, Sexton et al. [6] proposed cross-efficiency evaluation as an amplification of DEA and Doyle and Green [7] generalized their idea. The main idea in cross-efficiency evaluation is checking each DMU with the whole

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DMU's weights (instead of evaluation based on their own weights). The peer-evaluation assesses the performances of the DMUs not only in terms of their optimistic efficiencies, but also their cross-efficiencies computed using the weights determined by other peer DMUs.

Cross-efficiency evaluation has a good discrimination power among DMUs, it is used in various applications such as nursing homes efficiency evaluation (Sexton et al. [6]), preference voting and project ranking (Green et al. [8] and Wu et al. [9]), determining efficient operators and measuring the labor assignment in cellular manufacturing systems (Ertay and Ruan [10]), portfolio selection in Korean stock market (Lim et al. [11]) and so on.

Nevertheless, there are some problems in cross-efficiency evaluation. Perhaps the main problem is the existence of multiple optimal solutions for the weights resulted from the DEA model that leads to various efficiency scores (depending on the selection of weights). In order to overcome this problem, various secondary goals such as the aggressive and benevolent and neutral models were introduced ([6], [7], [12]). This goals are a potential adjustment which avoid the deduction of the cross-efficiency advantages. In the case of the benevolent model, for example, the idea is to identify optimal weights that maximize not only the efficiency of a particular DMU under evaluation but also the average efficiency of other DMUs. In the case of the aggressive model, one seeks weights that minimize the average efficiency of those other DMUs. Unlike the aggressive and benevolent formulations in cross-efficiency evaluation, the neutral DEA model determines one set of input and output weights for each DMU from its own point of view without being aggressive or benevolent to the other DMUs. Liang et al. [13] in their attempt to expand Doyle and Green's [7] models, suggested three various secondary goals from a benevolent perspective. Similarly, Wang and Chin [14] proposed some alternative secondary goal models, but they replaced the target efficiency of each DMU from the ideal point 1 to CCR efficiency. Wu et al. [9] suggested a DEA cross-efficiency evaluation based on Pareto improvement. Wang and Chin [12] proposed a neutral DEA model for cross-efficiency evaluation to avoid the difficulty in making a choice between the two different formulations. Ramon et al. [15] proposed a new approach to cross-efficiency evaluation. Wang et al. [16] provide four more neutral DEA models for cross-efficiency evaluation from the perspective of multiple criteria decision analysis (MADA). Jahanshahloo et al. [17] suggested the selection of symmetric weights as a secondary goal in cross-efficiency evaluation. Jahanshahloo and Fallahnejad [18] proposed a method for obtaining a unique solution for cross efficiency and then try to rank all DMUs including efficient (extreme or non-extreme) and inefficient units.

In this article, first ideal and anti-ideal DMUs are introduced according to Wang et al.'s [16] description. Then we suggest that each DMU be just evaluated by the ideal and anti-ideal DMUs instead of being evaluated by other DMUs.

The rest of this article is organized as follows: In the next section is presented a concise point out of the cross-efficiency evaluation and its main formulations and secondary goals. The proposed method is introduced in Section 3 and also the mentioned method is illustrated by some numerical examples in Section 4. Finally, Section 5 is assigned to the conclusions.

## 2 Cross-Efficiency

Suppose that there are a set of n Decision Making Units (DMUs), and each $DMU_j$ $(j=1,...,n)$, using different m inputs, produces different s outputs which are respectively determined by

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$x_{ij}(i=1,...,m)$ and $y_{ij}(r=1,...,s)$. To assess each $DMU_k(k=1,...,n)$, the efficiency score $E_{kk}$ performance can be calculated by the input-oriented CCR multiplier model as follows :

$$
\begin{aligned}
&Max \quad E_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk}, \\
&s.t. \\
&\qquad \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1,\ldots,n, \\
&\qquad v_{ik} \ge 0, \qquad\qquad i = 1,...,m, \\
&\qquad u_{rk} \ge 0, \qquad\qquad r = 1,\ldots,s.
\end{aligned}
\tag{1}
$$

Let $v_{ik}^{*}(i=1,...,m)$ and $u_{rk}^{*}(r=1,...,s)$ is the optimal solution of LP model above, then $E_{kk}^{*} = \sum_{r=1}^{s} u_{rk}^{*} y_{rk}$, shows the CCR-efficiency of $DMU_k$, which resulted from self-evaluation. However, $E_{kj}^{*} = \sum_{r=1}^{s} u_{rk}^{*} y_{rj} / \sum_{i=1}^{m} v_{ik}^{*} x_{ij}$ shows the cross-efficiency of $DMU_j$ resulted from peer-evaluation using $DMU_k(k=1,...,n;k \neq j)$. The corresponding model for each DMU is solved and as a result n series of input and output's weights for n DMU are computed. Each DMU has (n-1) a cross-efficiency in addition to one CCR efficiency. This efficiencies constitute an $n \times n$ matrix that $E_{kj}$ is an entry in row k and in column j which is called the cross-efficiency matrix. The cross-efficiency score for $DMU_j$ can be calculated as the average of $E_{kj}^{*}(k=1,...,n)$ in equation (2).

$$E_j = \frac{1}{n} \sum_{k=1}^{n} E_{kj}^{*} \tag{2}$$

Due to the fact that model (1) may have multiple optimal solutions. To resolve the non-uniqueness issue of input and output weights, Secondary goals to solve the problem of multiple optimal weights were introduced to examine the solution among the multiple optimal solutions on the basis of a criterion. For the first time, Sexton et al. [6] discussed the benevolent and aggressive models. Doyle and Green [7] exhibited another form of benevolent and aggressive formulas which are practically used more frequently.

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$$
\begin{aligned}
&Min \ \sum_{r=1}^{s} u_{rk} \Big( \sum_{j=1, j\neq k}^{n} y_{rj} \Big) \\
&s.t. \\
&\sum_{r=1}^{s} y_{ik} \Big( \sum_{j=1, j\neq k}^{n} x_{ij} \Big) = 1, \\
&\sum_{r=1}^{s} u_{rk} y_{rk} - E^{*}_{kk} \sum_{i=1}^{m} y_{ik} x_{ik} = 0, \\
&\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} y_{ik} x_{ij} \leq 0, \quad j = 1, \ldots, n, \\
&v_{ik} \geq 0, \quad \text{i} = 1, \ldots, m, \\
&u_{rk} \geq 0, \quad r = 1, \ldots, s.
\end{aligned}
\tag{3}
$$

and

$$
\begin{aligned}
&Max \ \sum_{r=1}^{s} u_{rk} \Big( \sum_{j=1, j\neq k}^{n} y_{rj} \Big) \\
&s.t. \\
&\sum_{r=1}^{s} y_{ik} \Big( \sum_{j=1, j\neq k}^{n} x_{ij} \Big) = 1, \\
&\sum_{r=1}^{s} u_{rk} y_{rk} - E^{*}_{kk} \sum_{i=1}^{m} y_{ik} x_{ik} = 0, \\
&\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} y_{ik} x_{ij} \leq 0, \quad j = 1, \ldots, n, \\
&v_{ik} \geq 0, \quad \text{i} = 1, \ldots, m, \\
&u_{rk} \geq 0, \quad r = 1, \ldots, s.
\end{aligned}
\tag{4}
$$

Model (3) and (4) intend to select optimal weights which in addition to preserving the efficiency of the unit under evaluation, successively decrease and increase other DMUs' cross-efficiency. These two models select optimal weights from two different views so that two different ranking methods in cross-efficiency evaluation are accrued. Wang and Chin [12] proposed a neutral DEA model for cross-efficiency evaluation to avoid the difficulty in making a choice between the two different formulations. The neutral DEA model determines one set of input and output weights for each DMU without being aggressive or benevolent to the others. Based upon this point of view, they construct the following neutral DEA model for the cross-efficiency evaluation of $DMU_k$:

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$$Max \quad \delta = \min_{r\in\{1,...,s\}} \quad \frac{u_{rk} y_{rk}}{\sum_{i=1}^{m} y_{ik} x_{ik}}$$

$$s.t.$$

$$\begin{aligned}
& E_{kk}^{*} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} y_{ik} x_{ik}}, \\
& E_{kj} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} y_{ik} x_{ij}} \le 1, \qquad j = 1,\ldots,n, \\
& v_{ik} \ge 0, \qquad\qquad \text{i} = 1,...,m, \\
& u_{rk} \ge 0, \qquad\qquad r = 1,\ldots,s.
\end{aligned} \tag{5}$$

Among multiple optimal weights, Model (5) selects the optimal weights which maximize the comparative efficiency of each output. In this method, the number of zero weights of output, decrease effectively. The weakness of this model is that, when we just face one output, it does not necessarily yield a unique optimal solution and indeed Model (5) changes into Model (1). To solve this problem, Wang et al. [16], by describing the ideal and anti-ideal virtual DMUs, suggested some neutral models on the basis of the distance from the ideal and anti-ideal virtual DMUs that are as follows:

$$Min \quad \mathrm{D}_{k}^{IDMU} = \sum_{r=1}^{s} u_{rk} (y_{r}^{\max} - y_{rk}) + \sum_{r=1}^{s} y_{ik} (x_{ik} - \mathrm{x}_{i}^{\min})$$

$$s.t.$$

$$\begin{aligned}
& \sum_{i=1}^{m} y_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} y_{ik} x_{ij} \le 0, \qquad j = 1,\ldots,n, \\
& v_{ik} \ge 0, \qquad\qquad \text{i} = 1,...,m, \\
& u_{rk} \ge 0, \qquad\qquad r = 1,\ldots,s.
\end{aligned} \tag{6}$$

and

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$$Max \quad D_k^{ADMU} = \sum_{r=1}^{s} u_{rk}(y_{rk} - y_r^{\min}) + \sum_{r=1}^{s} y_{ik}(x_i^{\max} - x_{ik})$$

$$\begin{aligned} &st. \\ &\sum_{i=1}^{m} y_{ik} x_{ik} = 1, \\ &\sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\ &\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} y_{ik} x_{ij} \le 0, \quad j = 1,\ldots,n, \\ &v_{ik} \ge 0, \quad i = 1,\ldots,m, \\ &u_{rk} \ge 0, \quad r = 1,\ldots,s. \end{aligned} \tag{7}$$

In model (6), each DMU seeks a set of input and output weights to minimize its distance from IDMU while keeping its optimistic efficiency unchanged. In model (7), each DMU looks for a set of input and output weights to maximize its distance from ADMU while remaining its optimistic efficiency unchanged. In models (6) and (7), what does this mean for the minimize its distance from IDMU and the maximize its distance from ADMU.

## 3 Proposed Method

Data envelopment analysis uses a linear programming problem so that it can select weights which are the best possible weights for the units under evaluation. When the best weights are chosen for a DMU, we can use these weights for other DMUs as well. In the new method, however, we aim for a cross-efficiency evaluation of DMUs by the use of the optimal weights of ideal and anti-ideal virtual DMUs. To this end, we first introduce ideal and anti-ideal virtual DMUs according to Wang et al.'s [16] description as follows:

**Definition 3.1.** Whenever by consuming the least input the maximum output is generated, we can obtain the ideal virtual DMU. In addition, if this virtual DMU generates the least output by consuming the maximum input, we call it anti-ideal DMU. We show the ideal and anti-ideal DMUs by IDMU and ADMU, respectively.

Given the description above, the IDMU's inputs and outputs can be determined as follows:

$$x_{iI} = \min_j \{x_{ij}\}, \quad i = 1,\ldots,m, \quad j = 1,\ldots,n, \tag{8}$$

$$y_{rI} = \max_j \{y_{rj}\}, \quad r = 1,\ldots,s, \quad j = 1,\ldots,n. \tag{9}$$

The ADMU's inputs and outputs can be computed as follows:

$$x_{iA} = \max_j \{x_{ij}\}, \quad i = 1,\ldots,m, \quad j = 1,\ldots,n, \tag{10}$$

$$y_{rA} = \min_j \{y_{rj}\}, \quad r = 1,\ldots,s, \quad j = 1,\ldots,n. \tag{11}$$

Now, we evaluate the ADMU and IDMU units by the following models, respectively:

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$$
\begin{aligned}
&Max \quad \sum_{r=1}^{s} u_{rI}^{j} y_{rI} \\
&s.t. \\
&\qquad \sum_{i=1}^{m} v_{iI}^{j} x_{iI} = 1, \\
&\qquad \sum_{r=1}^{s} u_{rI}^{j} y_{rj} - E_{jj}^{*} \sum_{i=1}^{m} v_{iI}^{j} x_{ij} = 0, \\
&\qquad \sum_{r=1}^{s} u_{rI}^{j} y_{rj} - \sum_{i=1}^{m} v_{iI}^{j} x_{ij} \le 0, \quad j = 1,\ldots,n, \\
&\qquad v_{iI}^{j} \ge 0, \qquad \text{i} = 1,\ldots,m, \\
&\qquad u_{rI}^{j} \ge 0, \qquad r = 1,\ldots,s.
\end{aligned}
\tag{12}
$$

and

$$
\begin{aligned}
&Max \quad \sum_{r=1}^{s} u_{rA}^{j} y_{rA} \\
&s.t. \\
&\qquad \sum_{i=1}^{m} v_{iA}^{j} x_{iA} = 1, \\
&\qquad \sum_{r=1}^{s} u_{rA}^{j} y_{rj} - E_{jj}^{*} \sum_{i=1}^{m} v_{iA}^{j} x_{ij} = 0, \\
&\qquad \sum_{r=1}^{s} u_{rA}^{j} y_{rj} - \sum_{i=1}^{m} v_{iA}^{j} x_{ij} \le 0, \quad j = 1,\ldots,n, \\
&\qquad v_{iA}^{j} \ge 0, \qquad \text{i} = 1,\ldots,m, \\
&\qquad u_{rA}^{j} \ge 0, \qquad r = 1,\ldots,s.
\end{aligned}
\tag{13}
$$

That $E_{jj}^{*}$ is CCR efficiency score of $DMU_j$. The $v_{iI}^{j*}$ and $u_{rI}^{j*}$ are the multiple optimal input's and output's weights of $DMU_j$ in the evaluation of ideal unit respectively and $v_{iA}^{j*}$ and $u_{rA}^{j*}$ are the multiple optimal input's and output's weights of $DMU_j$ in the evaluation of anti-ideal unit, respectively.

We suppose that Models (12) and (13) are solved *n* times and the optimal solutions $v_{iI}^{j*}, u_{rI}^{j*}$, $v_{iA}^{j*}$, $u_{rA}^{j*}$ $(\forall i, r, j)$ are gained, respectively. Then cross-efficiency $DMU_j$, by the use of the ideal and anti-ideal optimal weights, are computed as follows:

$$E_{Ij} = \sum_{r=1}^{s} u_{rI}^{j*} y_{rj} \Big/ \sum_{i=1}^{m} v_{iI}^{j*} x_{ij}, \quad j = 1,\ldots,n, \tag{14}$$

$$E_{Aj} = \sum_{r=1}^{s} u_{rA}^{j*} y_{rj} \Big/ \sum_{i=1}^{m} v_{iA}^{j*} x_{ij}, \quad j = 1,\ldots,n. \tag{15}$$

The new cross-efficiency score is gained as follows:

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$$E_j = \left(E_{Ij} + \left(1 - E_{Aj}\right)\right)/2\,,\quad j = 1,\ldots,n. \tag{16}$$

In this method, a new secondary goal is discussed such that it prevents the existence of multiple optimal solutions. We can put this secondary goal in a neutral secondary goal category, not in aggressive and benevolent one, because it does not have any relation to the cross-efficiency of other DMUs. Indeed, among the multiple optimal solutions, Model (12) and (13) select a solution based on the ideal and anti-ideal virtual DMUs' evaluation.

This method is capable of giving a different ranking in the evaluation of the efficiency of DMUs. In the suggested method, the number of computed cross-efficiency is *2n*, whereas in other methods, cross-efficiency is computed $n^2$ times. Therefore, when the number of DMUs increases, the use of this method in DMUs ranking becomes computationally economical.

## 4 Numerical Examples

In this section, we illustrate the performance of the proposed approach and compare it with those of some of the classical DEA cross-efficiency procedures by using a couple of small data sets that have frequently appeared in the related literature.

**Example 4.1.** Consider the following famous problem to illustrate the proposed method. Sexton et al. [6] considered a case of six nursing homes whose input and output data for a given year and ideal and anti-ideal DMUs are reported in Table 1, in which the input and output variables are defined as follows:

$x_1$: staff hours per day, including nurses, physicians, etc.

$x_2$: supplies per day, measured in thousands of dollars.

$y_1$: total medicare-plus medicaid-reimbursed patient days (0000).

$y_2$: total privately paid patient days (0000).

**Table 1** Data and CCR efficiency scores

| Nursing homes (DMUs) | $x_1$ | $x_2$ | $y_1$ | $y_2$ | CCR Efficiency |
|---|---|---|---|---|---|
| A | 1.5 | 0.2 | 1.4 | 0.35 | 1 |
| B | 4 | 0.7 | 1.4 | 2.1 | 1 |
| C | 3.2 | 1.2 | 4.2 | 1.05 | 1 |
| D | 5.2 | 2 | 2.8 | 4.2 | 1 |
| E | 3.5 | 1.2 | 1.9 | 2.5 | 0.9775 |
| F | 3.2 | 0.7 | 1.4 | 1.5 | 0.8674 |
| IDMU | 1.5 | 0.2 | 4.2 | 4.2 | |
| ADMU | 5.2 | 2 | 1.4 | 0.35 | |

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The average cross-efficiencies of the six nursing homes obtained by different alternative models are presented in Table 2. As you can see, the ranks of $DMU_A$, $DMU_B$, $DMU_C$ and $DMU_D$ are equal and cannot be distinctive, so we rank the DMUs by the use of cross-efficiency. Table 2 exhibits cross-efficiencies scores of the 6 home nurses resulted from Models (3), (4), (5), (6), (7) and the proposed method with their ranks. You can see that $DMU_A$ gained the first rank in Models (3), (4), (5), (7) and in the suggested method, whereas it gained the sixth rank in Model (6). Also, $DMU_A$ allocates itself the second rank in Models (3), (5), (7) and the suggested method, whereas it gained the first rank in Models (4), (6). Please note that $DMU_A$ and $DMU_D$ have the same rank in Doyle and Green's [2] benevolent model, however, distinguishing one from others is impossible, whereas the suggested method is able to rank these kinds of DMUs uniquely.

**Table 2** Cross-efficiencies score of the six nursing homes by different models and their rankings

| DMU | Model(3) | Model(4) | Model (5) | Model(6) | Model(7) | Proposed method |
|---|---|---|---|---|---|---|
| A | 0.7639(1) | 1.0000(1) | 0.9519 (1) | 0.7496(2) | 0.8120(1) | 0.6889(1) |
| B | 0.7004(3) | 0.9773(2) | 0.9190 (4) | 0.7004(4) | 0.7144(3) | 0.4744 (6) |
| C | 0.6428(5) | 0.8580(4) | 0.8314 (5) | 0.6630(5) | 0.6428(5) | 0.5858 (4) |
| D | 0.7184(2) | 1.0000(1) | 0.9500 (2) | 0.7684(1) | 0.7176(2) | 0.6450(2) |
| E | 0.6956(4) | 0.9758(3) | 0.9215 (3) | 0.7383(3) | 0.6984(4) | 0.6261 (3) |
| F | 0.6081(6) | 0.8570(5) | 0.8017(6) | 0.6228(6) | 0.6209(6) | 0.5000 (5) |

**Table 3** Data and CCR efficiency score

| DMU | $x_1$ | $x_2$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | CCR Efficiency |
|---|---|---|---|---|---|---|---|
| 1 | 17.02 | 5 | 42 | 45.3 | 14.2 | 30.1 | 1 |
| 2 | 16.46 | 4.5 | 39 | 40.1 | 13 | 29.8 | 1 |
| 3 | 11.76 | 6 | 26 | 39.6 | 13.8 | 24.5 | 0.982 |
| 4 | 10.52 | 4 | 22 | 36 | 11.3 | 25 | 1 |
| 5 | 9.5 | 3.8 | 21 | 34.2 | 12 | 20.4 | 1 |
| 6 | 4.79 | 5.4 | 10 | 20.1 | 5 | 16.5 | 1 |
| 7 | 6.21 | 6.2 | 14 | 26.5 | 7 | 19.7 | 1 |
| 8 | 11.12 | 6 | 25 | 35.9 | 9 | 24.7 | 0.961 |
| 9 | 3.67 | 8 | 4 | 17.4 | 0.1 | 18.1 | 1 |
| 10 | 8.93 | 7 | 16 | 34.3 | 6.5 | 20.6 | 0.954 |
| 11 | 17.74 | 7.1 | 43 | 45.6 | 14 | 31.1 | 0.983 |
| 12 | 14.85 | 6.2 | 27 | 38.7 | 13.8 | 25.4 | 0.801 |
| IDMU | 3.67 | 3.8 | 43 | 45.6 | 14.2 | 31.1 | |
| ADMU | 17.74 | 8 | 4 | 17.4 | 0.1 | 16.5 | |

**Example 4.2.** The following problem will be useful to illustrate our method.
In this example, we use the data set in Shang and Sueyoshi [19] that are presented in Table 3 together with both the ideal and anti-ideal DMUs and the CCR efficiency scores. Note that DMUs 1, 2, 4, 5, 6, 7, 9 gain the same rank, therefore, we start to rank them by the use of the cross-efficiency resulted from the mentioned models. The cross-efficiency score of 12 manufacturing systems and their rankings are illustrated in Table 4. For instance, you can see that $DMU_4$ gained the first rank in Model (6) and in the suggested method, the second rank in Model (4), (5), (7), and the third rank in Model (3). Also, $DMU_5$ received the second rank in Model (7) and in the suggested method, the first rank in Models (3), (4),(7) and the third rank

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in Model (5). Note that $DMU_{12}$ gained the tenth rank in all methods except that Model (5) and $DMU_9$ have the worst performance in all methods.

**Table 4** Cross-efficiencies score of the 12 manufacturing systems by different models and also their rankings

| DMU | Model(3) | Model(4) | Model (5) | Model(6) | Model(7) | Proposed method |
|---|---|---|---|---|---|---|
| 1 | 0.8483(2) | 0.9550(5) | 0.8968(3) | 0.8752(3) | 0.8588(3) | 0.5305(7) |
| 2 | 0.8391(4) | 0.9355(6) | 0.8781(5) | 0.8721(4) | 0.8482(4) | 0.5385(5) |
| 3 | 0.7767(5) | 0.9245(8) | 0.8731(6) | 0.8007(5) | 0.8287(5) | 0.5501(4) |
| 4 | 0.8441(3) | 0.9812(2) | 0.9343(2) | 0.9212(1) | 0.8945(2) | 0.6162(1) |
| 5 | 0.8668(1) | 0.9770(3) | 0.9570(1) | 0.9160(2) | 0.9398(1) | 0.5841(2) |
| 6 | 0.7273(8) | 0.9556(4) | 0.8421(7) | 0.7506(9) | 0.7355(9) | 0.4955(11) |
| 7 | 0.7581(6) | 0.9879(1) | 0.8792(4) | 0.7799(6) | 0.7739(6) | 0.5337(6) |
| 8 | 0.7243(9) | 0.9308(7) | 0.8226(8) | 0.7644(7) | 0.7261(8) | 0.5508(3) |
| 9 | 0.5638(12) | 0.7487(12) | 0.6204(12) | 0.5942(12) | 0.4867(12) | 0.4887(12) |
| 10 | 0.6178(11) | 0.8147(10) | 0.7213(11) | 0.6621(11) | 0.6120(11) | 0.5056(9) |
| 11 | 0.7472(7) | 0.9077(9) | 0.8167(9) | 0.7580(8) | 0.7507(7) | 0.5139(8) |
| 12 | 0.6675(10) | 0.7734 (11) | 0.7367(10) | 0.6968(10) | 0.7127(10) | 0.5000(10) |

## 5 Conclusions

Until now, cross-efficiency evaluation has been an important applied method for the comparison and ranking of DMUs. The main problem in cross-efficiency evaluation is the existence of multiple optimal solutions for the gained weights from DEA model that leads to various efficiency scores. In this article, we suggested that it is better for each DMU's cross-efficiency to be just computed by the use of ideal and anti-ideal virtual DMUs' evaluation so that by describing the new secondary goal, we can prevent the existence of the multiple optimal solutions and describe a new efficiency score as well.

As can be seen in the examples above, this method is able to give a different ranking in the evaluation of the efficiency of DMUs. Another advantage of this method in comparison with the previous methods is lees computation needed to determine the cross-efficiency score. If the number of DMUs is large, using this method for ranking seems beneficial.

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