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# Geometric Programming Problem with Trapezoidal Fuzzy Variables

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**Abstract** Nowadays Geometric Programming (GP) problem is a very popular problem in many fields. Each type of Fuzzy Geometric Programming (FGP) problem has its own solution. Sometimes we need to use the ranking function to change some part of GP to the linear one. In this paper, first, we propose a method to solve multi-objective geometric programming problem with trapezoidal fuzzy variables, then we use ranking function to sole one type of fuzzy geometric programming problem called Monomial Geometric Programming problem with respect to Trapezoidal fuzzy numbers. At the end, with an example we show how FGP is used in our real life. To illustrate the method, we use numerical examples.

**Keyword:** Fuzzy Geometric Programming, Monomial Geometric Programming, Ranking Function, Trapezoidal Fuzzy numbers, Fuzzy shortest path, Platform index $T$ , Multi-Objective function.

## 1 Introduction

Nowadays the fuzzy set theory is popular in many fields. One of these is fuzzy geometric programming problem that is applied in engineering system and science management. For the first time, Tanaka, *et al*. proposed the fuzzy mathematical programming problem [1]. Afterwards, Zimmermann [2] proposed the first formulation of Fuzzy Linear Programming(FLP) problem.

Geometric programming is a type of nonlinear programming problem. In fact geometric programming is mostly an extension comprehension of linear programming applications and is naturally classified in many types of nonlinear sets. In the beginning, GP was applied in engineering and sciences. At first the most applications of geometric programming were in chemical and mechanical engineering, statistics and probability, economics, wireless networking, etc. [3-5].

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Nowadays, fuzzy geometric programming (FGP) is very important in fields such as engineering, statistics, economic and management. Many authors in such fields have been focusing on the different types of FGP to propose and develop the best method to solve the FGP problem and find the optimal solution and optimal value.

Zadeh [6] presented the parameters by fuzzy numbers. Lin and Chern [7] proposed the algorithm for the type of network flow that arc lengths have fuzzy numbers to finding the most vital arcs. Hernandes, *et al*. [8] considered a generic algorithm by using any fuzzy numbers ranking index on the decision-maker.

In [9], Mahdavi, *et al*. proposed ranking order between fuzzy numbers, then improved a dynamic programming approach for the fuzzy shortest chain problem. For more information see [10]. Amit Kumar [11] proposed a new method based on which the decision maker obtains the fuzzy shortest path between each node and source node and use ranking function for comparing paths.

There are several kinds of fuzzy geometric programming problems. For solving each of them first we should classified them. In this paper, a method is proposed to find the optimal fuzzy solution and optimal fuzzy value of multi-objective geometric programing problem with respect to trapezoidal fuzzy numbers. Cao [12] defined the method to solve primal geometric programing problem and the variables on trapezoidal fuzzy numbers. In this paper we extend this method to compute the optimal solution of fuzzy multi-objective geometric programming problem.

In one example, we use ranking function to change fuzzy numbers to real numbers [10,13] to compare them and then computing the optimal solution. Also in one of the examples we illustrate that we deal with fuzzy geometric programming problem in a real world scenario.

The rest of this paper is organized as follows: Section 2 reviews some basic definitions of fuzzy numbers and trapezoidal fuzzy numbers arithmetic and introduces ranking function and platform index $T$. In Section 3 we review some main theorems and definitions and introduce monomial GP and multi-objective GP. In Section 4 we deal with presenting a method to solve multi-objective geometric programming and illustrate it by using examples and explain some applications of FGP. The conclusions are discussed in Section 5.

## 2 Preliminaries

In this section we review some basic and necessary definitions and theorems.

**Definition 2.1.** [12] The subset $\tilde{A}$ in set $X$ defined as $\tilde{A} = \{(\mu_{\tilde{A}}(x), x) \mid x \in X\}$, where $\mu_{\tilde{A}}(x)$ is a real number which belongs to the closed interval [0,1]. $\mu_{\tilde{A}}(x)$ is degree of membership $x$ in $\tilde{A}$ and $\mu_{\tilde{A}}(x)$

$$\mu_{\tilde{A}} : X \to [0,1]$$
$$x \to \mu_{\tilde{A}}(x)$$

is a membership function in fuzzy set $\tilde{A}$ .

**Definition 2.2.** We denote the trapezoidal fuzzy number as $\tilde{A} = (a^-, a^+, \underline{a}, \overline{a})$ and show the set of all trapezoidal fuzzy numbers with $F(\mathbb{R})$.

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**Definition 2.3.** [12] Fuzzy number $\tilde{A}=(a^-,a^+,\underline{a},\overline{a})$ is said to be a trapezoidal fuzzy number if its membership function defined as follows

$$\mu_{\tilde{A}}(x)=\begin{cases}0 & ; \quad x<a^-,x>\overline{a}\\ \dfrac{x-a^-}{a^+-a^-} & ; \quad a^-\le x\le a^+\\ 1 & ; \quad a^+\le x\le \overline{a}\\ \dfrac{\overline{a}-x}{\overline{a}-\underline{a}} & ; \quad \underline{a}\le x\le \overline{a}\end{cases}$$

**Definition 2.4.** The equation

$$\begin{aligned} &Min \quad g_0(x)=\sum_{k=1}^{J_0}a_{0k}\prod_{l=1}^{m}x_l^{\alpha_{0kl}} \qquad (1)\\ &s.t. \quad g_i(x)=\sum_{k=1}^{J_i}a_{ik}\prod_{l=1}^{m}x_l^{\alpha_{ikl}}\lesssim 1, \qquad (1\le i\le p)\\ &\quad x>0, \end{aligned}$$

called the fuzzy posynomial geometric programming problem, where $x=(x_1,x_2,...,x_m)^T$ is a m-dimensional variable vector and $g_i(x);(0\le \mathrm{i}\le \mathrm{p})$, is fuzzy posynomial of $x$, i.e., $a_{ik}>0$ is a constant, $a_{ikl}$ is an arbitrary real number, $x_l$ is a positive variable and $\lesssim$ is fuzzified version of $\le$ .

**Definition 2.5.** [12] Let $\tilde{A}=(a^-,a^+,\underline{a},\overline{a})$ and $\tilde{B}=(b^-,b^+,\underline{b},\overline{b})$ are two trapezoidal fuzzy numbers. The arithmetic operations properties on trapezoidal fuzzy numbers are denoted as follows:

1) $\tilde{A}+\tilde{B}=(a^-+b^-,a^++b^+,\underline{a}+\underline{b},\overline{a}+\overline{b})$,
2) $c\ge 0,c\in\mathbb{R};c\tilde{A}=(ca^-,ca^+,c\underline{a},c\overline{a})$,
3) $c<0,c\in\mathbb{R};c\tilde{A}=(c\overline{a},c\underline{a},ca^+,ca^-)$,
4) $\tilde{A}-\tilde{B}=(a^--\overline{b},a^+-\underline{b},\underline{a}-b^+,\overline{a}-b^-)$,
5) $\tilde{A}>0\ ,\tilde{B}>0;\ \tilde{A}\times\tilde{B}=\left(a^-b^-\ ,a^+b^+,\underline{a}\,\underline{b}\ ,\overline{a}\overline{b}\right)$.

**Definition 2.6.** [12] Assume that $x_{li}^{\ -}$ and $x_{li}^{\ +}$ are left and right endpoints of an interval, then, for $\tilde{x}_{li}$ arbitrary belongs to a closed value interval $[x_{li}^{\ -},x_{li}^{\ +}]$ whose degree of accomplishment is determined by

$$\mu_{\tilde{\phi}_i}(\tilde{x}_{li})=\begin{cases}0 & ; \quad x_{li}\le x_{li}^-\\ \left(\dfrac{x_{li}-x_{li}^-}{x_{li}^+-x_{li}^-}\right)^n & ; \ x_{li}^-<x_{li}\le x_{li}^+\\ 1 & ; \quad x_{li}>x_{li}^+\end{cases} \qquad (2)$$

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Where $n$ denotes a natural number.

**Definition 2.7.** [12] Assume that $\tilde{x}_i = (\tilde{x}_{1i}, \tilde{x}_{2i}, \ldots, \tilde{x}_{ni})(\mathrm{i} = 1, 2, ..., \mathrm{N})$. Divide the set of real natural numbers $\{1, 2, ..., \mathrm{n}\}$ into two distinct groups of odd numbers and even numbers, mutually exclusive subsets $T(-)$ and $T(+)$. To each partition associate a binary multi-index $T = (T_1, T_2, ..., T_m)$ defined by $t_p = \begin{cases} 0 & ; \quad p \in T(+) \\ 1 & ; \quad p \in T(-) \end{cases}$. We call $T$ a platform index.

## 2.1 Ranking Function

A current method for comparing fuzzy numbers is to use ranking function (see [14,15]).

**Definition 2.8.** [10] (Ranking Function) We call $\mathcal{R} : F(\mathbb{R}) \to (\mathbb{R})$ a ranking function that maps fuzzy numbers of $F(\mathbb{R})$ into real numbers $\mathbb{R}$ by natural ordering.

Suppose that $\tilde{A}_1, \tilde{A}_2 \in F(\mathbb{R})$, we define the orders with respect to ranking function $\mathcal{R}$ as follows:

$\tilde{A}_1 > \tilde{A}_2 \quad iff \quad \mathcal{R}\left(\tilde{A}_1\right) > \mathcal{R}\left(\tilde{A}_2\right)$,

$\tilde{A}_1 < \tilde{A}_2 \quad iff \quad \mathcal{R}\left(\tilde{A}_1\right) < \mathcal{R}\left(\tilde{A}_2\right)$,

$\tilde{A}_1 = \tilde{A}_2 \quad iff \quad \mathcal{R}\left(\tilde{A}_1\right) = \mathcal{R}\left(\tilde{A}_2\right)$.

**Note**. It is obvious that $\mathcal{R}$ is linear function such that $\mathcal{R}\left(c\tilde{A}_1 + \tilde{A}_2\right) = c\mathcal{R}\left(\tilde{A}_1\right) + \mathcal{R}\left(\tilde{A}_2\right)$, where $c \in \mathbb{R}$.

**Remark 2.1.** [11,16] Suppose that $\tilde{A} = (a^-, a^+, \underline{a}, \overline{a})$ is a trapezoidal fuzzy number. Ranking function is defined on $\tilde{A}$ as follows

$$\mathcal{R}\left(\tilde{A}\right) = \frac{a^- + a^+ + \underline{a} + \overline{a}}{4}.$$

## 3 Main Definition and Theorem

**Definition 3.1.** We call

$$Min \quad g_0^{(j)}\left(\tilde{x}\right) = \sum_{k=1}^{J_0^{(j)}} a_{0k}^{(j)} \prod_{l=1}^{m} \tilde{x}_l^{\alpha_{0kl}^{(j)}}, \qquad \left(1 \le j \le n\right) \tag{3}$$

$$s.t. \quad g_i\left(\tilde{x}\right) = \sum_{k=1}^{J_i} a_{ik} \prod_{l=1}^{m} \tilde{x}_l^{\alpha_{ikl}} \le \beta, \qquad \left(1 \le i \le p\right)$$

$$\tilde{x} > \tilde{0},$$

a multi-objective geometric programming with trapezoidal fuzzy variables where $\tilde{x} = \left(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n\right)^T$ signifying a m-dimensional fuzzy variable vector, $\tilde{\beta} \cong \beta \times \tilde{1}$, $\beta = \pm 1$,

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$\tilde{x}_i = \left(x_i^-, x_i^+, \underline{x}_i, \overline{x}_i\right)$ and $\tilde{1} = \left(1^-, 1^+, \underline{1}, \overline{1}\right)$ are trapezoidal fuzzy numbers, $g_0^{(j)}(\tilde{x})$ and $g_i(\tilde{x})$ are fuzzy posynomial of $\tilde{x}$ and $\alpha_{ikl}$ is an arbitrary real number.

**Theorem 3.1.** Suppose that the multi-objective FGP problem with trapezoidal fuzzy variables are the same as in (3), then it can be turned into a multi-objective FGP with a platform index $T$.

$$Min \quad g_0^{(j)}\left(z(T)\right) = \sum_{k=1}^{J_0^{(j)}} a_{0k}^{(j)} \prod_{l=1}^{m} \left(z_l(T)\right)^{\alpha_{0kl}^{(j)}}, \qquad (1 \le j \le n) \tag{4}$$

$$s.t. \quad g_i\left(z(T)\right) = \sum_{k=1}^{J_i} a_{ik} \prod_{l=1}^{m} (z_l(T))^{\alpha_{ikl}} \le 1, \qquad (1 \le i \le p)$$

$$z(T) > 0.$$

Also (3) contains an optimal solution with trapezoidal fuzzy variables, which is equal to (4) containing an optimal solution depending on a platform index $T$.

**Proof**. Similarly to the proof of Theorem 7.8.2 chapter 7 in [12], (3) is turned into

$$Min \quad \sum_{k=1}^{J_0^{(j)}} a_{0k}^{(j)} \prod_{l=1}^{m} \left(z_l(T)\right)^{\alpha_{0kl}^{(j)}}, \qquad (1 \le j \le n)$$

$$s.t. \quad \sum_{k=1}^{J_i} a_{ik} \prod_{l=1}^{m} \left(z_l(T)\right)^{\alpha_{ikl}} \le 1, \qquad (1 \le i \le p)$$

$$z(T) > 0, \qquad (1 \le l \le m)$$

such that (4) can be found.
Since (3) is equivalent to (4), a parameter optimal solution to (4) depending on a platform index $T$ is equivalent to an optimal trapezoidal fuzzy one to (3).

**Definition 3.2.** We call

$$Min \quad \tilde{a}_0 \prod_{j=1}^{m} \tilde{x}_j^{\tilde{\alpha}_{0j}} \tag{5}$$

$$s.t. \quad \tilde{a}_i \prod_{j=1}^{m} \tilde{x}_j^{\tilde{\alpha}_{ij}} \le \tilde{t}_i, \qquad (1 \le i \le n)$$

$$\tilde{x} > \tilde{0},$$

a fully fuzzy monomial geometric programming, where $\tilde{x} = \left(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n\right)^T$ is a m-dimensional fuzzy variable vector. Here $\tilde{x}_i = \left(x_i^-, x_i^+, \underline{x}_i, \overline{x}_i\right)$ is a trapezoidal fuzzy number, $\tilde{a}_i > \tilde{0}$ is a coefficient fuzzy number, $\tilde{t}_i > \tilde{0}$ is a fuzzy number and $\tilde{\alpha}_{ij}$ is an arbitrary fuzzy number.

**Theorem 3.2.** Any fuzzy posynomial geometric programming (1) can change into fuzzy convex programming.
**Proof.** Let $x_j = e^{z_j}$ for $1 \le j \le m$. Then

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$$\sum_{k=1}^{J_i} a_{ik} \prod_{j=1}^{m} x_j^{\alpha_{ikj}} = \sum_{k=1}^{J_i} a_{ik} e^{\sum_{j=1}^{m} z_j \alpha_{ikj}} = H_i(z), \quad 0 \le i \le n. \tag{6}$$

From [17] the conclusion of the theorem holds.

**Remark 3.1.** [12] Any fuzzy posynomial geometric programming problem (1) can turn into a monomial fuzzy geometric programming problem.

**Theorem 3.3.** [18] Any monomial fuzzy posynomial geometric programming (5) can turned into a fuzzy linear programming with the optimal solution

$$Min \quad \ln \tilde{a}_0 + \sum_{j=1}^{m} \tilde{a}_{ij} \tilde{z}_j, \tag{7}$$

$$s.t. \quad \ln \tilde{a}_i + \sum_{j=1}^{m} \tilde{a}_{ij} \tilde{z}_j \le \ln \tilde{t}_i, \quad (1 \le i \le n),$$

$$\tilde{z}_j > \tilde{0}.$$

## 4 The applications of geometric programming

In this section we investigate two different types of geometric programming problem with trapezoidal fuzzy variables and numbers, and glimpse the algorithm considering to solve them, then by presenting an example we show that how GP can be used in real world scenario.

### 4.1 Multi-Objective GP

For a fuzzy multi-objective geometric programming (3) with trapezoidal fuzzy variables, the objective functions can be weighted either before or after nonfuzzification or nonfuzzified. Here we study the algorithm presented by [12] for FGP and then we solve the fuzzy multi-objective GP by utilizing this algorithm.

A. Non-fuzzification steps
The steps of nonfuzzification applied to problem (3) is as follows:
Let $\tilde{x}_l = (\tilde{x}_{l1}, \tilde{x}_{l2}, \ldots, \tilde{x}_{lp})^T$ be a trapezoidal fuzzy variable where $\tilde{x}_{li} = (x_{li}^-, x_{li}^+, \underline{x}_{li}, \overline{x}_{li})$ for $(1 \le l \le m, 1 \le i \le p)$. Arbitrary $\tilde{x}$ belongs to the closed value interval $[x_{li}^-, x_{li}^+]$, so we choose the degree of accomplishment in determined by membership function (2), then by $\tilde{\phi}_i(\tilde{x}_{li}) \ge \gamma$, we conclude:
$x_{li} - x_{li}^- \ge \sqrt[n]{\gamma}(x_{li}^+ - x_{li}^-) \Rightarrow x_{li} = x_{li}^- + \sqrt[n]{\gamma}(x_{li}^+ - x_{li}^-)$.

I) For a given trapezoidal fuzzy variable $\tilde{x}_l$, we partition natural number set $\{1, 2, ..., m\}$ to three exclusive sets:
   1. For $l = 1, 2, \ldots, M$ and any $i$ :

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$$\tilde{x}_{li} \to x_{li}^{-} + \sqrt[n]{\gamma}\left(x_{li}^{+} - x_{li}^{-}\right) + \frac{\underline{x}_{li} + \overline{x}_{li}}{2}.$$

2. For $l = M+1, M+2, \ldots, 2M$ and any $i$ :

$$\tilde{x}_{li} \to \begin{cases} x_{li}^{-} + \sqrt[n]{\gamma}\left(x_{li}^{+} - x_{li}^{-}\right) + \underline{x}_{li} \;, & j_l = 0, \\ x_{li}^{-} + \sqrt[n]{\gamma}\left(x_{li}^{+} - x_{li}^{-}\right) - \overline{x}_{li} \;, & j_l = 1. \end{cases}$$

3. For $l = 2M+1, 2M+2, \ldots, 3M$ and any $i$ :

$$\tilde{x}_{li} \to \begin{cases} x_{li}^{-} + \sqrt[n]{\gamma}\left(x_{li}^{+} - x_{li}^{-}\right) - \underline{x}_{li} \;, & j_l = 0, \\ x_{li}^{-} + \sqrt[n]{\gamma}\left(x_{li}^{+} - x_{li}^{-}\right) + \overline{x}_{li} \;, & j_l = 1. \end{cases}$$

Nonfuzzify variable $\tilde{x}_l$ .

II) Let $z_{li} = x_{li}^{-} + \sqrt[n]{\gamma}\left(x_{li}^{+} - x_{li}^{-}\right)$. Then under the platform index $T$, $z_l(T) = (z_{li} + \sum_{i=1}^{3M} x_{li}^{*} \frac{m}{3M})$, where $x_{li}^{*}$ is $\frac{\underline{x}_{li} + \overline{x}_{li}}{2}$ , $\pm \underline{x}_{li}$ or $\pm \overline{x}_{li}$ .

III) Replace $z_l(T)$ instead of $\tilde{x}_l$, then the trapezoidal fuzzy variables (3) is turned into a determined variable (4).

IV) Figure out a sufficient solution to problem (4).

B) Direct primal algorithm

Nonfuzzify (3) into (4) before weighting the objective function in (4).

Give weight to $u_j \left(1 \le j \le n\right)$ for $n$ objective function.

Now in (4), $g_0^{*}\left(z(T)\right) = u_1 g_0^{(1)}\left(z(T)\right) + u_2 g_0^{(2)}\left(z(T)\right) + \ldots + u_n g_0^{(n)}\left(z(T)\right)$, where $u_j$ is a weighted factor satisfying $0 \le u_j \le 1 \left(1 \le j \le n\right)$, and $u_1 + u_2 + \ldots + u_n = 1$.

Put $g_0^{*}\left(z(T)\right)$ for $n$ objective function in (4), then it changes to a single objective parameter geometric programming

$$\begin{aligned} &Min \quad g_0^{*}\left(z(T)\right) \qquad (8) \\ &s.t. \quad g_i\left(z(T)\right) \le 1, \qquad \left(1 \le i \le n\right) \\ &\qquad z(T) > 0. \end{aligned}$$

And calculate it.

**Note**. A fuzzy acceptable solution of (3) behaves similar to single objective geometric programming problem with respect to platform index $T$, because the primal algorithm approach to an approximate acceptable solution.

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**Example 4.1.** Find

$$Min \quad g_0^{(1)}(\tilde{x}) = \tilde{x}_1 - 1 \quad , \quad Min \quad g_0^{(2)}(\tilde{x}) = (\tilde{x}_2 - 1)^2 \tag{9}$$

$$s.t. \quad (\tilde{x}_1 - 1)(\tilde{x}_2 - \frac{1}{4}) \le \tilde{1} ,$$

$$\tilde{x}_1 \le \tilde{2} ,$$

$$\tilde{x}_2 \le \frac{\tilde{5}}{4} ,$$

$$\tilde{x}_1 , \tilde{x}_2 > \tilde{0} .$$

Where $\tilde{x}_1 = (x_1^-, x_1^+, \underline{x}_1, \overline{x}_1)$ and $\tilde{x}_2 = (x_2^-, x_2^+, \underline{x}_2, \overline{x}_2)$ are two trapezoidal fuzzy variables and $\tilde{1} = (1,1,0,0)$, $\tilde{2} = (2,2,0,0)$ and $\frac{\tilde{5}}{4} = (\frac{5}{4}, \frac{5}{4}, 0, 0)$ are trapezoidal fuzzy numbers.

Suppose that $\tilde{x}_1$ and $\tilde{x}_2$ are trapezoidal fuzzy data as:

$\tilde{x}_1$ : $1)(x_1^-, x_1^+, 1, 0)$ $3)(x_1^-, x_1^+, 2, 1)$ $5)(x_1^-, x_1^+, 1.5, 2)$
$7)(x_1^-, x_1^+, 0.5, 0.5)$ $9)(x_1^-, x_1^+, 2, 2)$ $11)(x_1^-, x_1^+, 1, 2)$

$\tilde{x}_2$ : $2)(x_2^-, x_2^+, 1.5, 1.2)$ $4)(x_2^-, x_2^+, 0, 1)$ $6)(x_2^-, x_2^+, 1, 2)$
$8)(x_2^-, x_2^+, 0, 1.5)$ $10)(x_2^-, x_2^+, 0, 0)$ $12)(x_2^-, x_2^+, 2, 1)$

$\tilde{x}_1$ and $\tilde{x}_2$ should belong to close value intervals $[1,2]$ and $[\frac{1}{4}, \frac{5}{4}]$ respectively. From equation (2) put $n = 1$, then

$$\begin{cases} \dfrac{x_1 - x_1^-}{x_1^+ - x_1^-} = \dfrac{x_1 - 1}{2 - 1} = x_1 - 1 = \mu_{\tilde{\phi}_1}(\tilde{x}_1) \ge \gamma_1 \\ \dfrac{x_2 - x_2^-}{x_2^+ - x_2^-} = \dfrac{x_2 - \frac{1}{4}}{\frac{5}{4} - \frac{1}{4}} = x_2 - \frac{1}{4} = \mu_{\tilde{\phi}_2}(\tilde{x}_2) \ge \gamma_2 \end{cases} \quad \Rightarrow \quad \begin{cases} x_1 \ge \gamma_1 + 1 \\ x_2 \ge \gamma_2 + \dfrac{1}{4} \end{cases}$$

It equals to $\tilde{x}_1 : x_1 \ge \gamma_1 + 1$ and $\tilde{x}_2 : x_2 \ge \gamma_2 + \frac{1}{4}$, where $\gamma_1, \gamma_2 \in [0,1]$. So we have

$\tilde{x}_1$ : $1)(\gamma_1 + 1, 1, 0)$ $3)(\gamma_1 + 1, 2, 1)$ $5)(\gamma_1 + 1, 1.5, 2)$
$7)(\gamma_1 + 1, 0.5, 0.5)$ $9)(\gamma_1 + 1, 2, 2)$ $11)(\gamma_1 + 1, 1, 2)$

$\tilde{x}_2$ : $2)\ (\gamma_2 + \frac{1}{4}, 1.5, 1.2)$ $4)(\gamma_2 + \frac{1}{4}, 0, 1)$ $6)\ (\gamma_2 + \frac{1}{4}, 1, 2)$
$8)\ (\gamma_2 + \frac{1}{4}, 0, 1.5)$ $10)\ (\gamma_2 + \frac{1}{4}, 0, 0)$ $12)(\gamma_2 + \frac{1}{4}, 2, 1)$

Now we should divide the data to three groups and also by the Definition (2.7), $j_l = 1$ for odd numbers and $j_l = 0$ for even numbers.

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1. Numbers 1,4,7 and 10
   $\gamma_1+\frac{3}{2},\gamma_2+\frac{3}{4},\gamma_1+\frac{3}{2},\ \gamma_2+\frac{1}{4}$
2. Numbers 2,5,8 and 11
   $\gamma_2+\frac{7}{4},\gamma_1-1,\gamma_2+\frac{1}{4},\ \gamma_1-1$
3. Numbers 3,6,9 and 12
   $\gamma_1+2,\gamma_2-\frac{3}{4},\gamma_1+3,\ \gamma_2-\frac{3}{4}$ .

So $\gamma_1$ and $\gamma_2$ can be nonfuzzified as follows

$$\gamma_1 \to \frac{\gamma_1+\frac{3}{2}+\gamma_1+\frac{3}{2}+\gamma_1-1+\gamma_1-1+\gamma_1+2+\gamma_1+3}{6}=\frac{6\gamma_1+6}{6}=\gamma_1+1,$$

$$\gamma_2 \to \frac{\gamma_2+\frac{3}{4}+\gamma_2+\frac{1}{4}+\gamma_2-\frac{7}{4}+\gamma_2+\frac{1}{4}+\gamma_2-\frac{3}{4}+\gamma_2-\frac{3}{4}}{6}=\frac{6\gamma_2+\frac{6}{4}}{6}=\gamma_2+\frac{1}{4}.$$

Replace $\gamma_1+1$ and $\gamma_2+\frac{1}{4}$ instead of $\tilde{x}_1$ and $\tilde{x}_2$ in (9) respectively. So we have

$$\begin{aligned} &Min \quad g_0^{(1)}(\gamma_1)=\gamma_1 \quad , \quad Min \quad g_0^{(2)}(\gamma_2)=\gamma_2^2 \\ &s.t. \quad \gamma_1\gamma_2 \le 1, \\ &\qquad \gamma_1 \le 1, \\ &\qquad \gamma_2 \le 1, \\ &\qquad \gamma_1,\gamma_2 > 0. \end{aligned}$$

This is a multi-objective geometric programming problem corresponding to platform index $T$. By using an objective-weighted method, an objective function is changed into $g_0(\tilde{x})=u_1 g_0^{(1)}(\tilde{x})+u_2 g_0^{(2)}(\tilde{x})$. We attain $g_0(\alpha)=u_1\gamma_1+u_1\gamma_2^2$. Suppose that $u_1=u_2=\frac{1}{2}$, so

$$\begin{aligned} &Min \quad \frac{1}{2}\gamma_1+\frac{1}{2}\gamma_2^2 \\ &s.t. \quad \gamma_1\gamma_2 \le 1, \\ &\qquad \gamma_1 \le 1, \\ &\qquad \gamma_2 \le 1, \\ &\qquad \gamma_1,\gamma_2 > 0. \end{aligned}$$

The optimal solution of above equation is $\gamma_1=1,\gamma_2=1$ and the optimal value is $g_0(\gamma)=1$, so optimal solution to (9) is $x_1^*=2$ and $x_2^*=\frac{5}{4}$ .

The optimal solution to (9) can be changed, because as we explained $u_1+u_2=1$, so $u_1$ and $u_2$ can change.

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### 4.2 Monomial Geometric Programming

As we know, monomial GP is the type of geometric programming problem. Here by utilizing Theorem (3.2) and Theorem (3.3), first we turn the GP problem into linear one and then by keeping numbers in fuzzy form and using Definition (2.5) we find the equations and easily solve them.

**Example 4.2**. Solve the fully fuzzy monomial geometric programming problem

$$\begin{aligned} \textit{Min} \quad & \tilde{x}_1^{\tilde{2}}\tilde{x}_2^{\tilde{1}}, \\ \textit{s.t.} \quad & \tilde{x}_1^{\tilde{1}}\tilde{x}_2^{\tilde{2}} = e^{\tilde{8}}, \\ & \tilde{x}_1^{\tilde{2}}\tilde{x}_2^{\tilde{3}} = e^{\tilde{13}}, \\ & \tilde{x}_1, \tilde{x}_2 > \tilde{0}. \end{aligned}$$

Where $\tilde{1} = (1,2,1,1), \tilde{2} = (2,3,1,2), \tilde{3} = (3,4,2,3), \tilde{8} = (8,18,3,8), \tilde{13} = (13,25,5,13)$ and $\tilde{0} = (0,0,0,0)$.

**Solution.**
By applying Theorem (3.3), we obtain

$$\begin{aligned} \textit{Min} \quad & \tilde{2} \times \tilde{z}_1 + \tilde{1} \times \tilde{z}_2, \\ \textit{s.t.} \quad & \tilde{1} \times \tilde{z}_1 + \tilde{2} \times \tilde{z}_2 = \tilde{8} \times 1, \\ & \tilde{2} \times \tilde{z}_1 + \tilde{3} \times \tilde{z}_2 = \tilde{13} \times 1, \\ & \tilde{z}_1, \tilde{z}_2 > \tilde{0}. \end{aligned}$$

Now substitute the values $\tilde{1},\tilde{2},\tilde{3},\tilde{8},\tilde{13}$ and variables $\tilde{z}_1 = \left(z_1^-, z_1^+, \underline{z}_1, \overline{z}_1\right)$ and $\tilde{z}_2 = \left(z_2^-, z_2^+, \underline{z}_2, \overline{z}_2\right)$ in the above equation.

$$\begin{aligned} \textit{Min} \quad & (2,3,1,2) \times \left(z_1^-, z_1^+, \underline{z}_1, \overline{z}_1\right) + (1,2,1,1) \times \left(z_2^-, z_2^+, \underline{z}_2, \overline{z}_2\right), \\ \textit{s.t.} \quad & (1,2,1,1) \times \left(z_1^-, z_1^+, \underline{z}_1, \overline{z}_1\right) + (2,3,1,2) \times \left(z_2^-, z_2^+, \underline{z}_2, \overline{z}_2\right) = (8,18,3,8), \\ & (2,3,1,2) \times \left(z_1^-, z_1^+, \underline{z}_1, \overline{z}_1\right) + (3,4,2,3) \times \left(z_2^-, z_2^+, \underline{z}_2, \overline{z}_2\right) = (13,25,5,13), \\ & \tilde{z}_1, \tilde{z}_2 > \tilde{0}. \end{aligned}$$

Now by computing the above equation, we have:

$$\begin{aligned} \textit{Min} \quad & (2z_1^- + z_2^-, 3z_1^+ + 2z_2^+, \underline{z}_1 + \underline{z}_2, 2\overline{z}_1 + \overline{z}_2), \\ \textit{s.t.} \quad & z_1^- + 2z_2^- = 8, \\ & 2z_1^+ + 3z_2^+ = 18, \\ & \underline{z}_1 + \underline{z}_2 = 3, \\ & \overline{z}_1 + 2\overline{z}_2 = 8, \\ & 2z_1^- + 3z_2^- = 13, \\ & 3z_1^+ + 4z_2^+ = 25, \\ & \underline{z}_1 + 2\underline{z}_2 = 5, \end{aligned}$$

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$$2\overline{z}_1 + 3\overline{z}_2 = 13,$$
$$\tilde{z}_1, \tilde{z}_2 > \tilde{0}.$$

By solving equations, we obtain $z_1^- = 2, z_1^+ = 3, \underline{z}_1 = 1, \overline{z}_1 = 2$ and $z_2^- = 3, z_2^+ = 4, \underline{z}_2 = 2, \overline{z}_2 = 3$, so $\tilde{z}_1 = (2,3,1,2), \tilde{z}_2 = (3,4,2,3)$ and $\left(2z_1^- + z_2^-, 3z_1^+ + 2z_2^+, \underline{z}_1 + \underline{z}_2, 2\overline{z}_1 + \overline{z}_2\right) = (7,17,3,7)$.

Now $\tilde{x}_1^* = \left(e^2, e^3, e^1, e^2\right)$ and $\tilde{x}_2^* = \left(e^3, e^4, e^2, e^3\right)$ are the optimal solutions of the problem and the optimal value is $\left(e^7, e^{17}, e^3, e^7\right)$.

### 4.3 Network Flow Problem

Network is a graph containing finite number of nodes and arcs, here arcs length has numerical value. Network flow problem is usual in engineering and management. In real world scenarios, the arcs length value is not crisp. Many authors proposed different solutions for this type of problem. In this example we study the computation of the fuzzy shortest path between source node and the destination node in network flow problem presented by [19] and [11].

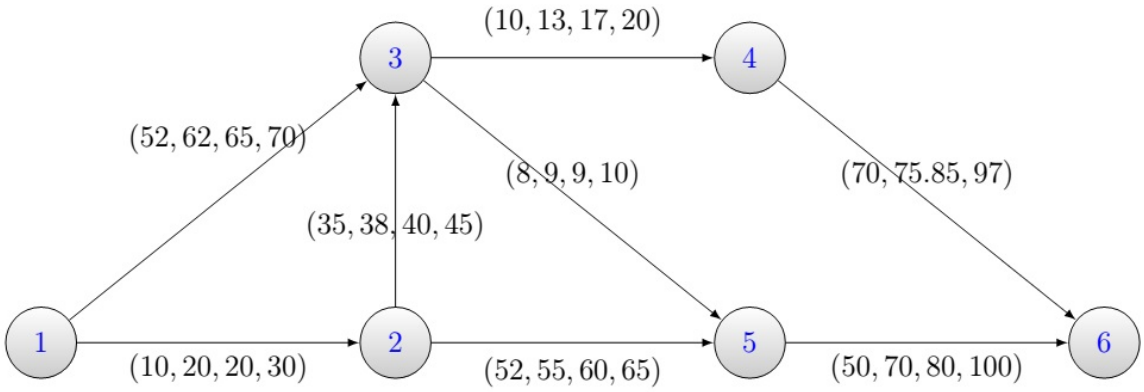



**Fig. 1** Network flow with fuzzy arc lengths [12]

Both methods approached to $1 \to 2 \to 3 \to 5 \to 6$ and node 6 can labeled as $\left[(103,137,149,185), 5\right]$.

### 5 Conclusions

In this paper we introduced multi-objective geometric programming with trapezoidal fuzzy variables, then we applied this method to solve FGP corresponding to the platform index $T$, which was used on primal geometric programming with trapezoidal fuzzy variables by Cao [12]. We construed a fully fuzzy monomial geometric programming problem with trapezoidal fuzzy numbers by keeping fuzzy numbers in whole solution. Also by an example we described FGP in a part of daily life in network flow problem.

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